Review of the relation between galaxy clusters and the constraining parameters of the ΛCDM model.

Ammar Emad, STEM High School for Boys – 6th of October.

Abdelhakim Mohamed-Fahmy, STEM High School for Boys – 6th of October.

Omar Hassan, STEM High School for Boys – 6th of October.

Mentor: Mustafa Mohammed, STEM High School for Boys – 6th of October.

Abstract

We reviewed the relationship between clusters of galaxies and the ΛCDM (Lambda Cold Dark Matter) model of the universe. Since the formation and the characteristics of galaxy clusters support the theory behind the ΛCDM model, we discussed the formation process of galaxy clusters, their morphological characteristics, different observational techniques, and the parameters of the universe. We also mention methods from other cosmological probes that agree with galaxy cluster observations, hence, validating the ΛCDM as the standard model of the universe.

I. Introduction

The ΛCDM model serves as the prevailing cosmological framework that describes the evolution and structure of the universe, as it has developed over the decades, providing a fundamental understanding of our cosmos. The historical development and theoretical foundations of the ΛCDM model can be traced back to the early 20th century with Einstein's general relativity solutions as the model's theoretical bedrock. Over time, advancements in observational astronomy, along with theoretical insights, have led to the refinement and formulation of the model. Dark matter, a non-luminous and elusive form of matter, is a key assumption of the ΛCDM model which dominates gravitational interactions on cosmic scales, providing gravitational scaffolding for the formation of galaxies and large-scale structures. Furthermore, dark energy, an enigmatic form of energy that permeates the universe, is incorporated into the model to account for the observed accelerated expansion of the universe denoted by the Hubble constant H_o. Cosmological parameters characterize the properties of the universe, providing intrinsic quantities that encode information about its fundamental attributes. Determining the cosmological parameters of the universe such as the total density parameter and the dark energy parameter, decides the composition of matter in the universe and determines the dynamics within the universe. Determining these cosmological parameters applies to many theories of gravity and particle physics. However, to constrain these parameters we use galaxy clusters, large-scale cosmic structures bound by forms of gravity, which serve as laboratories for refining the model of the universe. Their abundance, spatial distribution, and mass measurements, measured through optical and x-ray surveys, are used to test and refine theoretical predictions, providing critical constraints that allow us to distinguish between different cosmological scenarios.

We start by examining the formation and evolution of galaxy clusters. Furthermore, we discuss observational methods that are used to infer data from clusters of galaxies, providing examples of some surveys, and satellites. Moving on, we describe the morphological characteristics of galaxy clusters, such as their shapes, and distribution. Finally, we define the cosmological parameters of the universe, and their implications, mentioning examples from other studies, and validating them through comparison with other cosmological probes.

II. GALAXY CLUSTER FORMATION AND EVOLUTION

This chapter explores the fascinating realm of galaxy cluster formation and evolution. We delve into the hierarchical structures that emerge from initial density fluctuations, the gravitational collapse and merging processes that shape these clusters, and the assembly and growth of these colossal objects. Additionally, we delve into the crucial roles played by dark matter and dark energy, unraveling their gravitational influences on structure formation and their effects on cosmic expansion and cluster dynamics.

A. Hierarchical structures formation

In this subsection, we explore three key aspects related to the formation and evolution of galaxy clusters: initial density fluctuations, gravitational collapse and mergers, and the assembly and mass growth of galaxy clusters. We then examine the process of gravitational collapse, in which gravity causes celestial bodies to attract matter toward their center of mass, leading to the formation of dense structures. Finally, we study the mass assembly and evolution of galaxy clusters, address the challenges of applying the models developed to smaller structures, and highlight efforts to stop to understand the relationship between ordinary matter and dark matter.

1) Initial density fluctuations

The probability distribution function (PDF) of the cosmological density fluctuations is an essential characteristic of the universe's large-scale bodies, like galaxy clusters. The random Gaussian distribution is what the PDF of the primordial density fluctuations responsible for the universe's current structures is assumed to obey in the standard picture of gravitational instability. The PDF stays Gaussian as long the density fluctuations are in the linear range. However, because of the substantial nonlinear mode coupling and the nonlocality of the gravitational dynamics, their PDF significantly departs from the original Gaussian form once they reach the nonlinear stage.[1]

2) Gravitational collapse and merge process

Gravitational collapse happens when the force of gravity of a celestial body attracts its matter to its center of mass. This process is crucial to the formation of new structures in the universe. This collapse creates highly dense structures such as galaxy clusters, stars, and planets from less dense structures. Collapse compression causes the temperature to rise until a thermonuclear fusion at the center of the star occurs. The collapse stops gradually as the pressure exerted by the center of the star balances the gravitational force. After this process, the star enters a state of dynamic equilibrium until the star’s energy is consumed. Then, the star re-enters this process to transform into a white dwarf. [2]

3) Mass assembly and growth of galaxy clusters

In the scale of galaxy clusters, the models made for other smaller structures are ineffective in describing these huge objects. There are many inconsistencies between the basic models and mechanisms and the empirical data. On a scale between 1 kpc and 1 Mpc, the connection between ordinary and dark matter is poorly understood. Scientists have created the GAMA(Galaxy And Mass Assembly) survey project to solve this problem. The goal of the GAMA survey is to offer the most comprehensive wide-area dataset for low- to intermediate-redshift galaxies that is now technologically feasible. This project is focused on three important keys: improving spectroscopic efficiency, improving spatial resolution, and increasing wavelength coverage. This project also challenges the current CDM paradigm by detecting the baryonic systematics of galaxy formation, measuring the dark matter halo mass function, and estimating the galaxy merger rates over five billion years by counting the observed close pairings. [3]

B. Role of dark matter and dark energy in the universe

We explore the role of dark matter and dark energy in the universe deeply, as they are two mysterious components that have a significant impact on the formation, dynamics, and expansion of structures. Dark matter, although invisible, exerts a gravitational influence on the formation of structures, such as galaxy clusters, which scientists can deduce by observing its effects on light. On the other hand, dark energy is a mysterious form of energy that affects the expansion of the universe.

1) Dark matter gravitational influence on structure formation

According to recent studies, the universe is composed of 5% of ordinary matter. The rest of the universe is composed of dark matter and dark energy. Dark matter is not really dark, but invisible. Though the dark matter is assumed to be transparent, it has mass. Scientists were able to calculate the mass of dark matter because of its gravitational influence. Scientists can calculate the amount of dark matter in a galaxy cluster by observing how gravity affects light. This is called gravitational lensing which provides how much mass in a cluster and where it is. Gravitational lensing makes the light from a single source take many paths, providing us with many images from different angles for one source. In terms of its overall contribution to the total mass and energy of the cosmos, scientists used variations in the cosmic microwave background, estimating that dark matter makes up around 27% of the universe. Dark matter's gravitational influence and use of gravitational lensing help scientists calculate its mass and location, contributing to our ongoing understanding of the universe's composition. [4]

2) Dark energy's effect on cosmic expansion and cluster dynamics

In the 1920s, many scientists observed that there are galaxies that move away from us. After revising data with the general theory of relativity, researchers concluded that the universe is expanding. In the 1980s, researchers announced that they observed cosmic expansion precisely and concluded that the rate of expansion is not constant. It suggests that an unidentified force is working against gravity to accelerate the expansion of the cosmos. This force is later called the dark energy. Although dark matter and dark energy are both invisible, they are so different from each other. Dark matter connects galaxies while dark energy pushes them away from each other. The scientists figured out the continuous expansion of the universe when they observed a group of supernovae called standard candles farther away from what should it be in a deaccelerating universe. Dark energy is estimated to account for 69% of the universe. While dark matter and dark energy cannot be seen, discoveries showed that these mysterious forces play pivotal yet opposing roles in shaping the large-scale structure and future expansion of our universe. [5]

III. OBSERVATIONAL METHODS AND DATA

Studying celestial bodies involves a collection of many observational methods and data sets. Classifying these objects depends on analyzing the received data. In this section, a review of the existing methods is provided, highlighting their significance in astrophysics research. Additionally, the challenges and limitations are addressed, emphasizing the importance of careful analysis and interpretation of these techniques.

A. Optical observations

Optical observations focus on analyzing the apparent characteristics of galaxies and the bodies within them. Imaging and morphological analysis of galaxies provide a comprehensive understanding of the formation, evolution, and visual classification of galaxies. Then, the discussion moves to the redshift survey analysis for the identification of galaxy clusters. Redshift surveys enable the determination of galaxies' velocities and distances, shedding light on the dynamics and distribution of galaxies across cosmic scales.

1) Imaging and morphological analysis of galaxies:

As galaxies have different morphological characteristics, analyzing their complex shapes and structures reflects the past, present, and future of the universe which helps study galaxies and galaxy clusters. Many schemes are used in identifying, many of which depend on morphological features such as the number of spiral arms, the number of nuclei, the size of the bulge, etc. Imaging galaxies depends on several methods whether on space or ground which will be discussed forward. Before the advancement of technology, all the data were taken and analyzed manually. While human analysis is mostly accurate, with the advancement of surveys, it would take a longer time to analyze data. In the galaxy zoo project, it took 3 years to analyze ~300,000 galaxies. Thus, researchers have developed machine learning algorithms to classify, for instance, galaxies at a faster rate that keeps up with modern surveys. Although using machine learning algorithms helps in saving time and effort, it has certain amounts of uncertainty which has been an ongoing research area to maximize the efficiency of these algorithms. [6]

2) Redshift surveys for galaxy cluster identification:

Redshift is a phenomenon that occurs when electromagnetic radiation emitted or reflected from an object tends to lose energy and shifts to the higher wavelength end of the spectrum. Redshifts are caused due to several factors. The Doppler effect is a phenomenon in which the wavelength of a wave is changed due to the movement of either the observer or the source of the radiation which can be computed using Equation 1. Where Z is the redshift, v is the recessional velocity, and c is the speed of light.

$$1 + z = \sqrt{\frac{c + v}{c - v}}$$

Equation 1: Relativistic redshift

However, considering relativistic effects changes the equation as it includes the time dilation caused by special relativity which is computed by equation 2.

$$z = \frac{v}{c}$$

Equation 2: Classical Doppler redshift

Where Z is the redshift, v is the recessional velocity, and c is the speed of light. Furthermore, initial redshifts beyond our galaxy were built on the Doppler effect. But with the discovery of the correlation between redshift and increasing distances of the galaxy, cosmological redshift was used to describe the redshift caused by the expansion of the universe rather than the physical velocity of the galaxies which is dependent on the cosmological scale factor. Moreover, a third type of redshift appears as a result of Einstein’s theory of general relativity which is caused by the time dilation due to gravitational wells. [7]

Redshift surveys measure the previous redshifts in a section of the sky to identify astronomical objects. By combining redshift surveys and the angular positions of the objects with the observed redshifts, a 3D distribution of matter in the universe can be mapped, helping in identifying large-scale structures of the universe. Generally, spectroscopy is used to measure the apparent wavelengths of the selected section of the sky, a field of science that studies visible light and electromagnetic radiations, analyzing the radiations to their spectra. Figure 1 shows a map of galaxies plotted by the data received from the 2MASS redshift survey. [7]

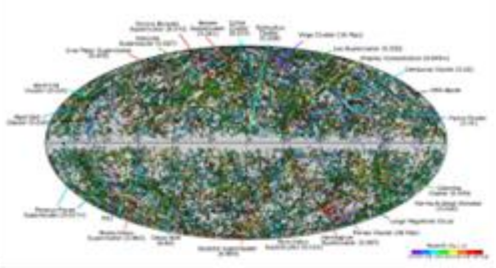

Figure 1: all 2MRS galaxies, spanning the entire redshift range covered by the survey (from z = 0 in purple to z = 0.08 in red.

B. X-ray observations

X-ray observations depend on unraveling the contents of the universe through studying X-ray wavelengths. We first discuss gas temperature, density, and X-ray emissions which provide an understanding of the hot ionized gasses within astronomical structures. Additionally, the determination of cluster mass and baryon content is discussed, emphasizing the role of X-ray observations in studying the gravitational effects and the distribution of matter within galaxy clusters.

1) Gas temperature, density, and X-ray emission:

X-ray emissions occur due to the hot gasses that fill the space between galaxies known as the intra-cluster medium (ICM for short). The ICM consists of ionized hydrogen and helium, along with heavier elements, which emit X-rays. Densities and temperatures of the ICM can be measured using X-ray surveys which by analysis give the distribution of gasses within the cluster. X-ray emissions assume the presence of cool cores within the cluster, characterized by a drop in X-ray emission and a peak in the surface brightness of the X-ray emitting gas, where the temperature of the gas is less than that of the surroundings. They appear due to cooling flows in which the gasses cool and fall towards the center of the cluster, releasing gravitational potential energy and heating up again. On the other hand, recent observations suggest that the cooling flows aren't as strong as they are thought to be, and there is still much debate on the regulations of gasses within clusters. X-ray observations have also revealed other structures within galaxy clusters, such as filaments, bubbles, and cavities. These structures are thought to be associated with various physical processes occurring within the cluster. For example, gas sloshing can create spiral patterns in the X-ray emission, while active galactic nuclei (AGN) feedback can create bubbles and cavities. Mergers between clusters can also produce shocks and turbulence that contribute to X-ray emission. In addition to thermal emission, X-ray observations have also detected non-thermal emission from clusters of galaxies which arise from high-energy particles accelerated by shocks in the cluster environment. The presence of non-thermal emission provides insights into the physical processes occurring in clusters such as particle acceleration and magnetic field amplification. [8]

2) Determination of cluster mass and baryon content:

One of the primary methods for determining cluster mass using X-ray observations is the hydrostatic equilibrium method. This approach relies on the assumption that the intracluster gas is in hydrostatic equilibrium within the gravitational potential of the cluster. In other words, the pressure gradient of the gas balances the gravitational force, resulting in a stable system. By measuring the X-ray luminosity and temperature of the gas, underlying gravitational potential can be inferred hence estimating the cluster mass. However, it is important to acknowledge the potential uncertainties associated with the hydrostatic equilibrium assumption. Deviations from spherical symmetry, the presence of non-thermal pressure sources, or the effects of dynamical processes can introduce systematic errors in mass determinations. Hence, alternative X-ray methods have been developed to address these issues. One such method is the X-ray mass proxy technique, which establishes empirical correlations between observable X-ray properties and cluster mass. By utilizing relationships between X-ray luminosity, temperature, and mass derived from statistical analyses of large cluster samples, estimating cluster masses can be done without relying on the assumption of hydrostatic equilibrium. These empirical relations provide valuable tools for determining mass in a more robust and model-independent manner. Additionally, using X-ray can determine the mass of the intracluster gasses themselves, providing more information on the gas and baryon content within clusters of galaxies.

C. Gravitational lensing

Gravitational lensing presents a technique for studying the bending of light by massive objects, revealing information about the distribution of mass in the universe. We first discuss the examination of the strong and weak lensing effects on background galaxies, highlighting how the distortion and magnification of light can be used to probe the mass distribution of foreground structures. Additionally, the discussion covers mass reconstruction and gravitational potential mapping, which involves the modeling and mapping of the gravitational lensing signal to infer the distribution of dark matter.

1) Strong and weak lensing effects on background galaxies:

Both types of gravitational lensing occur due to predictions made by the general theory of relativity in which massive objects can bend light passing nearby. Strong lensing occurs when massive systems, such as galaxy clusters, have high gravitational potential to bend light rays passing through them, forming arcs and multiple images as in Figure 2. These arcs give information about the mass distribution in galaxies. [9]

Figure 2: This HST image of the cluster SDSS J 1004+4112 shows arcs, multiply imaged galaxies, and a quadruply lensed quasar with a maximum image separation of

Weak gravitational lensing is the subtle distortion of the shapes of distant galaxies caused by the gravitational influence of foreground mass distributions. Unlike strong lensing, weak lensing does not produce multiple images but instead induces coherent, systematic shape distortions in the observed galaxy population. These distortions, known as shear, contain valuable information about the underlying mass distribution along the line of sight. [10]

2) Mass reconstruction and gravitational potential mapping:

Gravitational potential mapping involves inferring the distribution of gravitational potential, which is directly related to the matter distribution in the Universe. Mass reconstruction is a common tool for studying galaxy clusters. It's used to evaluate qualitatively the matching between the baryonic and dark matter distributions and to explore the possible existence of dark matter. The process involves creating mock strong lensing images, subtracting lens light, and reconstructing lensed images. gravitational potential mapping involves using mathematical tools to enhance parts of the gravity field and derive maps from the original gravity anomaly grid. These maps contain information about rock density depth and distribution of anomaly source rocks. By combining strong and weak lensing measurements, we can reconstruct the gravitational potential of massive objects and cosmic structures. [11][12]

D. Surveys and data sources

Surveys and data sources are a fundamental part of astrophysical research. In this part, we discuss several surveys and data sources that aided in the study of the cosmic microwave background and the discovery of astronomical bodies such as galaxies.

1) Planck satellite data and microwave observations:

Planck satellite, named after Max Planck, is sent to measure the cosmic microwave background over the whole sky. The satellite is composed of three main parts. The telescope focuses light onto the detectors which is done by 2 mirrors. It also protects the detectors from the glare of objects in the sky such as the moon or bright planets as shown in figure 3.

The satellite has two instruments. The low-frequency instrument, LFI, and the high-frequency instrument, HFI. Those are in a box under the mirrors with horns

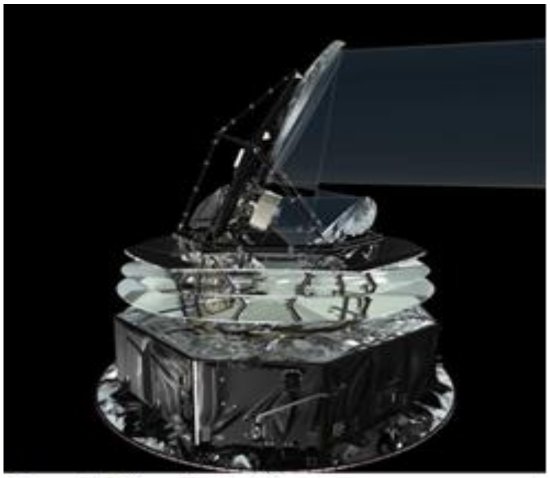

Figure 3: Planck satellite

that collect the radiation. Both instruments are cooled to 20K and 0.1K, preventing them from detecting their thermal glow. Both instruments look at different wavelengths. LFI looks at light with a wavelength of 10, 7, and 4 mm (corresponding to frequencies of 30, 45, and 75 GHz), while HFI looks at light with wavelengths between 3 and 0.3 mm (corresponding to frequencies between 100 and 900 GHz). [13]

2) *Large-scale surveys (e.g., SDSS, eROSITA, DES):*

Studying the universe depends on several observational methods. Large-scale surveys have provided insights into the composition of the universe. The Sloan Data Sky Survey, SDSS, main goal was to map the universe entirely. Since the project started, it has provided data on many celestial bodies such as galaxies, quasars, and stars. The extended Roentgen Survey with an Imaging Telescope Array, eROSITA, is an X-ray survey that aims to map the entire X-ray sky. X-rays provide a unique window into some of the most energetic processes in the universe, such as black holes, supernovae, and active galactic nuclei. The dark energy survey, DES, is a collaboration of scientists aiming to investigate the nature of dark energy by analyzing the mass distribution in the whole universe. These large-scale surveys have revolutionized our understanding of the universe by providing us with an unprecedented wealth of data. The sheer volume of observations collected by these projects would have been unimaginable just a few decades ago. [14][15][16]

IV. MORPHOLOGICAL CHARACTERISTICS

Morphological characteristics of galaxy clusters refer to the physical and structural properties that describe the appearance and arrangement of galaxies within these massive cosmic structures. Galaxy clusters come in different shapes. Some are Ellipsoidal, some are prolate, and others are oblate. Sometimes, smaller groups of galaxies can be found inside bigger ones. These clusters can also contain different types of galaxies.

A. Shapes of galaxy clusters

Galaxy clusters have various shapes, and their shapes are affected by many factors, like the dynamics of their galaxies, the gravitational forces, and the distribution of dark matter. The main shapes of galaxy clusters are ellipsoidal, prolate, and oblate clusters.

1) Ellipsoidal, prolate, and oblate clusters:

The shapes of galaxy clusters can be determined through observations, such as the distribution of their galaxies, gravitational lensing effects, and X-ray emissions from hot gas within the cluster. Ellipsoidal, prolate, and oblate clusters are geometric shapes that galaxy clusters take. These shapes are characterized by the distribution of galaxies and dark matter along different axes.

Ellipsoidal clusters are like elliptical clusters but have a more general ellipsoidal shape. This means that the cluster can be elongated or flattened in any direction, and its shape looks like an ellipsoid, which is a three-dimensional oval. The distribution of galaxies and dark matter is asymmetrical along all the three axes.

Prolate clusters are elongated along one axis, while the other two axes are relatively shorter. The galaxies and dark matter are concentrated along the long axis of the prolate cluster, which results in an obvious elongation in one direction.

Oblate clusters are flattened along one axis, with two longer axes. They have a disk-like appearance, and the galaxies and dark matter are concentrated in the central plane of the cluster, resulting in a flattened shape.

The difference between prolate and oblate clusters appears in the orientation of the elongation or flattening relative to the observer's line of sight. If the elongation or flattening is oriented along the line of sight, it appears as an elongation or flattening when viewed from Earth. Prolate clusters have their longest axis aligned with the line of sight, while oblate clusters have their shortest axis aligned with the line of sight.

2) The Influence of Cluster Mergers and Dynamical Processes on Galaxy Cluster Evolution:

Cluster mergers, also known as cluster collisions or interactions, happen when two or more galaxy clusters come together under the gravitational pull. These events are energetic and transformative. As clusters merge, galaxies, hot gas, and dark matter redistribute, changing the cluster's mass distribution and gravitational potential. An increasing amount of data has shown that many clusters are very complex systems. Optical analyses show that some clusters contain subsystems of galaxies suggesting that they are still in the phase of relaxation, sometimes after a phase of cluster merging. Simultaneously, the interaction generates shock waves in the intracluster medium (ICM), heating it and intensifying X-ray emissions. This heat affects gas dynamics and star formation in member galaxies. Cluster mergers also accelerate galaxy-galaxy interactions, changing galaxy morphology and star formation rates. [17]

In addition to mergers, the occurrence of dynamical processes plays essential roles in shaping galaxy clusters. Cluster gravitational potentials affect the orbits of member galaxies, impacting their positions and velocities. Galaxy-galaxy interactions, driven by close encounters, result in phenomena like tidal stripping of gas and stars, transforming galaxy morphology and inducing bursts of star formation. Dynamical friction further influences galaxy motion as it gradually slows massive galaxies and leads to their concentration within the cluster core.

B. Sizes and mass distribution

Galaxy clusters are giant cosmic entities, composed of galaxies, hot gas, and dark matter, and they are not static structures but dynamic structures that evolve on cosmic time scales. In this subsection, we discuss two important aspects of galaxy clusters: their size and extent, as well as their volume configuration and proportional relationships. We first discuss how galaxy clusters form and evolve and the importance of characterizing their characteristics and extent. Next, we explore the distribution of mass within these clusters and the scaling relationships that link the observable properties of a cluster to its total mass. Understanding these aspects has important implications for studies aimed at constraining cosmological parameters using galaxy clusters.

1) Characterizing cluster size and extent:

Galaxy clusters form by the gravitational merger of smaller clusters and groups. Major cluster mergers are the most energetic events in the universe since the Big Bang. Characterizing the size and extent of galaxy clusters is essential in our quest to understand these massive cosmic entities. Galaxy clusters, composed of galaxies, hot gas, and dark matter, are not static but dynamic structures that evolve over cosmic timescales. Characterizing the size and extent of galaxy clusters is essential in astrophysics. Galaxy clusters vary in size, shape, and composition. The virial radius (R_{vir}) is a fundamental measure that represents the boundary within which the cluster's gravitational forces balance cosmic expansion. Optical richness estimates cluster size based on galaxy counts within specific magnitude ranges. [18]

2) Mass profiles and scaling relations:

The mass profiles and scaling relations of galaxy clusters are crucial for understanding galaxy clusters as they provide insights into the distribution of mass within clusters and their properties.

Mass profiles are about the distribution of mass within a cluster. It tells us where most of the mass is and how it changes when moving away from the center.

Scaling relations are relations between the cluster's properties and its mass. It has been found that certain things, like a cluster's temperature, the number of galaxies in it, or its brightness, are linked to its mass. By measuring these characteristics, the cluster mass can be estimated. Well-calibrated scaling relations between the observable properties and the total masses of galaxy clusters are essential for understanding the physical processes that give rise to these relations. They are also crucial for studies that aim to constrain cosmological parameters using galaxy clusters. [19]

C. Substructures within galaxy clusters

Substructures provide insights into galaxy formation on intermediate scales, from galaxy groups and subgroups to filamentary morphologies and the large-scale cosmic web. Particular attention is paid to central dominant galaxies and brightest cluster galaxies, examining their distinctive yet pivotal roles as nexus points shaping respective local environments through gravitation interactions and evolutionary processes across cosmic history.

1) Galaxy groups and subclusters:

Galaxy groups and subclusters are important in understanding how galaxies come together and how gravity shapes the cosmos. These smaller structures also influence the properties and evolution of larger galaxy clusters. Galaxy groups are smaller gatherings of galaxies, they are part of the galaxy clusters where a few galaxies hang out together, bound by their mutual gravitational attraction. Galaxy groups usually contain fewer than 50 galaxies in a diameter of 1 to 2 megaparsecs (Mpc where 1 Mpc is approximately 3262000 light-years or 2×10^{19} miles). Their mass is approximately 10^{13} solar masses. Galaxy groups are less massive than galaxy clusters, and they have their special dynamics. [20]

Within a galaxy cluster, there can be multiple subclusters scattered around. Each subcluster is a small collection of galaxies, and they also can have their galaxy groups within them. Subclusters are part of the bigger cluster, and they contribute to the structure and dynamics of it.

2) Filamentary structures and cosmic web connections:

Filamentary structures and cosmic web connections are fundamental components of the large-scale structure of the universe. As they provide crucial insights into the distribution of matter on cosmic scales.

Filamentary Structures are long, thread-like formations that traverse the cosmos. They primarily consist of dark matter, gas, and galaxies. Dark matter plays an essential role in shaping these filaments as it acts as their gravitational scaffold. Along filamentary structures, galaxies align, forming a special pattern within the cosmic web. This web includes a complex network of filaments, vast voids, and massive galaxy clusters, shaping the very fabric of the universe.

The cosmic web itself is an astonishing structure. It connects all cosmic components, binding galaxy clusters, galaxies, and voids together. These filaments serve as the ways along which matter and galaxies travel. As it guides their movement through the cosmos. Understanding the cosmic web's architecture is crucial for understanding the universe's evolution and testing cosmological theories.

3) Central dominant galaxies and brightest cluster galaxies:

Central Dominant Galaxies (CDGs) and brightest cluster galaxies (BCGs) are two distinct classes of massive galaxies, as both play an essential role within their respective cosmic environment. CDGs are found in smaller galaxy groups, while

BCGs are located at the hearts of massive galaxy clusters.

CDGs are the most massive and luminous galaxies in their groups, they are often located near the group's center. They exert influence over the dynamics and evolution of their group as they impact the motion and interactions of other member galaxies. CDGs can manifest as elliptical or CD (central dominant) galaxies, with a more relaxed morphology. Their environments are relatively less crowded as their environments lack the extreme densities of galaxy clusters.

On the other hand, BCGs are found within massive galaxy clusters and are the most massive galaxies known. Most clusters and galaxy groups contain a giant elliptical galaxy in their centers that outshines and outweighs normal ellipticals. The origin of BCGs begins when multiple galaxies merge and grow within the gravitational environment of galaxy clusters. Over time, the largest and most luminous galaxy member emerges as the BCG, becoming a central and dominant presence within the cluster. [21]

D. Spatial distribution and clustering properties

Spatial distribution refers to how objects are distributed or arranged in space, specifically on large scales in the universe. It helps us understand the cosmic organization of galaxies, clusters, and other structures. Clustering properties provide insights into the degree and nature of objects' clustering in the universe. Two of the main clustering properties are the cluster-cluster correlation function and the power spectrum. They are analytical tools used in astrophysics and cosmology to study the large-scale distribution of matter, particularly galaxy clusters.

1) Large-scale spatial distribution of clusters:

The large-scale spatial distribution of galaxy clusters is a pivotal aspect of the universe's structure. These clusters are organized along large filamentary structures within the cosmic web. Furthermore, the large-scale distribution of clusters offers insights into the formation and evolution of cosmic structures. Numerical simulations based on these observations help us understand how clusters form and evolve over billions of years. In essence, the study of galaxy cluster distribution provides a vital glimpse into the universe's vast and complex architecture, deepening our comprehension of its fundamental principles.

2) Cluster-cluster correlation function and power spectrum:

The cluster-cluster correlation function, denoted as $\xi(r)$, assesses the clustering of galaxy clusters by measuring how the probability of finding clusters at different separations deviates from random distribution. Positive $\xi(r)$ values indicate clustering, while negative values signify anti-clustering. Studying $\xi(r)$ at various scales helps in determining cosmological parameters and the distribution of dark matter.

The power spectrum, represented as P(k), analyzes the matter's fluctuations on different spatial scales. By examining its shape, we gain insights into the nature of primordial density fluctuations, influencing the formation of structures like galaxy clusters. Observations of the power spectrum, whether from galaxy surveys or cosmic microwave background studies, provide crucial information for refining cosmological models and understanding the universe's fundamental properties.

3) Voids, superclusters, and cosmic variance effects

Voids are vast, empty regions in space without galaxies or clusters. They are integral to the cosmic web, influencing matter distribution and the universe's expansion. The study of voids helps scientists test cosmological models and understand cosmic dynamics. Superclusters are colossal, gravitationally bound structures containing multiple galaxy clusters. They are interconnected by filaments and represent the densest regions in the cosmos. Superclusters provide insights into gravitational interactions and the distribution of galaxies. Cosmic variance effects arise from limited observational sampling, causing fluctuations in galaxy and cluster distribution. These variations can introduce uncertainties in cosmological measurements.

V. CONSTRAINTS ON COSMOLOGICAL PARAMETERS

This section examines the constraints imposed on cosmological parameters. Through statistical methodologies like maximum likelihood estimation and Bayesian analysis, we unravel the intricacies of error estimation, unlocking insights into fundamental parameters shaping the cosmos by addressing systematic uncertainties and harnessing technological advancements.

A) Maximum likelihood estimation and Bayesian analysis

In scientific analysis, error estimation is a crucial part. Whenever any parameter is estimated, the error must be estimated too. Estimation is a process of obtaining model parameters from randomly distributed observations. With the advancement of technology during the last decades, two popular statistical tools are used for estimation. These two tools are Maximum likelihood estimation and Bayesian analysis. The key difference between these two tools is that the parameters for maximum likelihood estimation are fixed but unknown, while the Bayesian method parameters act as random variables with known prior distributions. Bayesian Estimation has results more accurate than MLE, but it is also more complex to compute than MLE. Overall, these two techniques are important to cosmological research in the means of error estimation. [22]

B. Systematic uncertainties

Statistical uncertainty is caused by stochastic processes. Fluctuations brought on by measurement errors are founded on a constrained number of observations. Because of this, a collection of observations from the same occurrence will differ from measurement to measurement. The statistical uncertainty is a gauge of the range of this variation. The term "statistical differences between" similar measurements of the same event were made twice are not connected.

1) Biases from sample selection and cluster misidentification:

Any propensity that inhibits a question from being considered objectively is referred to as bias. Bias in research happens when one outcome or response is favorably chosen or encouraged above others by systematic inaccuracy brought into sampling or testing.

In astronomy, there are three main types of bias: Malmquist bias, Bias frame, and Perturbative bias expansion. Malmquist bias is an effect in observational astronomy to the preferential detection of bright objects. A bias frame is essentially a zero-time length exposure. It can be used to adjust oblique frames when an exact time match with a light frame is unavailable. Perturbative bias expansion is a way to describe the clustering of galaxies on large scales by a finite set of coefficients of the expansion, called bias parameters. [23]

2) Controlling for astrophysical and instrumental uncertainties:

The main uncertainty is caused by the instrument itself. The measurements cannot be better than the instruments used to make them. This type of uncertainty can be improved using better instruments and newer technologies and techniques. Another type is systematic uncertainty. This can be an error in the setting up of the telescope. This error will not change during the trials. To change this error, fixing and revising every set-up piece is crucial. The last type is random uncertainty. To fix this error, making as many trials as possible is needed. [24]

C. Constraints on cosmological parameters

New technology and advancement in research led to the establishment of a precise cosmological model with cosmological parameters with an accuracy of around 10%. "Cosmological parameters" term refers to the parameters of the dynamics of our universe, such as the universe's curvature and expansion rate.

1) Density parameter (Ω_m):

The density parameter is the ratio of the density of matter and energy in the universe to the critical density (the density at which the universe will stop expanding). The density used in this ratio is the sum of the Byronic matter's density parameter, dark matter's density parameter, and dark energy's density parameter. When this ratio is less than one, the universe will be open and expanding forever. If more than one, the universe will stop expanding and recollapse. If this ratio is equal to one, the universe is flat: has enough energy to stop expanding but not enough for Recollapsing. To the accuracy of current studies, the universe has a density parameter equal to 1. [25]

2) Dark energy equation of state (w):

The equation of state parameter describes the rate at which the dark energy in the universe evolves. This equation is the rate of the pressure of the dark energy to its energy density. Dark energy has a negative pressure (tension). This feature is what allows the density to persist as the universe expands. To have a vacuum unchanged energy, w must be = -1 (The pressure is equal to the energy density in magnitude but opposite in direction. If w is greater than -1, the dark energy will decrease as the universe expands. Current experiments suggest that w is equal to -1. [26]

3) Hubble constant (H_0):

Hubble's constant is the measure of the speed of the objects moving away from us. Although it is named a constant it is not. Originally, Hubble measured this value as 500km/s. Now with the technology's availability, scientists argue about values between 68 and 74 km/s. The constant can be calculated using the distance to any object and its speed. The speed is measured using Doppler's effect. The precision of the value of this constant can lead us to discoveries in dark matter and energy. [27]

D. Friedmann's equations

Friedmann's equations are a set of equations that describe the expansion of the universe due to its matter content. Friedmann's equations provide models that describe many observational features in the universe. In the following paragraphs, we discuss the cosmological principle and constants, and we derive the first Friedmann equation.

1) The cosmological principle and Einstein's field equation:

The cosmological principle states that our universe is isotropic and homogeneous. The isotropic property of the universe means that on large scales the universe looks the same in all directions which leads to no preferred direction to look at. On the other hand, homogeneous means that the universe looks the same from all locations, on large scales, which means that there's no preferred location. The cosmological principle is a fundamental part of astrophysical research as it describes the main assumptions of the universe's structure. Einstein's field equations (EFE) are a set of equations that describe the interaction of gravitation. They consist of ten non-linear partial differential equations. In this paper, we will use the mostly negative matrix convention, hence, making a negative sign before the cosmological constant. Moreover, the energy-momentum tensor has negative signs on the spatial diagonal terms. However, using any convention will lead to deriving the same equations. Equation 3 represents the Einstein field equation. [26]

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Equation 2: Einstein's field equations.

Where R represents the Ricci tensor, T is the energy-momentum tensor, g is the FLRW metric, R is the Ricci scalar, c is the speed of light, G is the universal gravitational constant and Λ is the cosmological constant. [26]

2) The first Friedmann equation

Before deriving the equation, let’s define some constants and terms. The scale factor, a(t), is a function that measures the changing spatial scale of the universe over time. We will denote it as a and $\dot{a}$ as its first-time derivative and so on. Moreover, the cosmological constant, Λ, is a way to incorporate dark energy in the model. The cosmological constant acts as a form of energy and negative pressure that drives the expansion of the universe. The scalar constant, k, represents the geometric shape of the universe whether open, closed, or flat. Finally, ρ and p represent the density of the universe and the pressure as a function of time, respectively. To derive the first Friedmann equation, we take the 00 components of the EFE’s tensors, and we get equation 4.

$$\frac{(\dot{a}^2 + kc^2)}{a^2} = \frac{8\pi G\rho + \Lambda c^2}{3}$$

Equation 4: Friedmann's first equation.

3) Computing the cosmological parameters:

We can expand both sides of equation 4 and move the scalar constant to reach a form from which we can define some cosmological parameters. Since $H = \frac{\dot{a}}{a}$, we reach a simpler form as in equation 5. [27]

$$H^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{(kc^2)}{a^2}$$

Equation 5: An expanded form of Friedmann's equation.

Where H represents the Hubble constant. If we divide both sides of the equation by the square of Hubble’s constant, we define each term as a density parameter respectively as shown in equation 6. [26]

$$1 = \Omega_{M/R} + \Omega_{\Lambda} + \Omega_k$$

Equation 6: Friedmann's first equation.

Where $\Omega_{M/R}$ represents the total energy density stored in the non-relativistic matter and radiation, Ω Λ represents the dark energy parameter, and Ω_k represents the spatial curvature. [26]

To sum up, Friedmann's first equation can be altered to define the energy density parameters of the universe, enabling us to analyze the energy components of the universe.

VI. DISCUSSION AND IMPLICATIONS

In this section, we further discuss and make sense of the reviewed literature regarding the ΛCDM model, as well as compare it with other space probes. By examining the connection between the observed properties of galaxy clusters and the predictions of the ΛCDM model, we can confirm and evaluate the robustness of this standard model of the universe.

Additionally, we briefly mention the results of galaxy cluster studies with other space probes, such as observing supernovae, measuring cosmic microwave background anisotropy, and weak gravitational lensing.

A. Implications for the ΛCDM model

Validation of the ΛCDM model is essential to establish its credibility as a reliable framework for understanding the structure and evolution of the universe. By examining the observed properties of galaxy clusters against the predictions of the ΛCDM model, we can assess the degree of alignment and consistency between theory and observations. These implications not only strengthen our confidence in the ΛCDM model but also highlight areas of divergence and the need for further research into alternative models or extensions of the current framework.

1) Validating the standard cosmological paradigm:

The review demonstrates the alignment between the observed properties of galaxy clusters and the predictions of the ΛCDM model, thus providing validation for the standard cosmological paradigm. For instance, [27] combines high-resolution restimulations of cluster-sized dark haloes with semi-analytic galaxy formation modeling to compare the number. This approach allows a detailed description of the properties of galaxy clusters. Numerous studies have confirmed that the distribution, abundance, and scaling relations of galaxy clusters are consistent with the hierarchical growth of structure, as predicted by the ΛCDM model. Furthermore, the improved semi-analytic model utilized in the paper successfully reproduces observed luminosity functions, metallicities, color-magnitude relations of cluster galaxies, and the metal content of the ICM, supporting the notion that dark matter plays a role in the formation and evolution of galaxy clusters). The presence of dark energy, as represented by the cosmological constant Λ, drives the accelerated expansion of the universe. [27]

2) Constraining deviations and alternative models:

While the ΛCDM model has been remarkably successful in explaining various aspects of galaxy cluster properties, the literature review reveals certain discrepancies that warrant scientific investigation. [28] provides a detailed examination of these deviations. The researchers report significant discrepancies between the observed mass function or concentration-mass relation of galaxy clusters and the predictions of the ΛCDM model. One notable discrepancy lies in the concentration-mass relation predicted by the ΛCDM model, which does not consistently align with the observations obtained through X-ray studies of galaxy clusters. The expected relation suggests that higher mass clusters should have higher concentrations, but the observed data often deviate from this trend. These deviations challenge the precise predictions of the ΛCDM model and indicate the potential existence of additional factors influencing the concentration-mass relation. The presence of these discrepancies motivates further exploration of alternative models to explain the observed behavior of galaxy clusters. One avenue of investigation involves modified gravity theories, which propose modifications to the laws of gravity on large scales. These alternative models aim to account for the observed deviations by altering the gravitational interactions at play within galaxy clusters. Additionally, extensions of the ΛCDM framework, such as models incorporating additional forms of dark matter or modified dark energy dynamics, are also being considered to address the observed discrepancies. To fully comprehend the underlying causes of these deviations, in-depth investigations are necessary. It is crucial to assess whether these discrepancies arise from limitations in our current understanding and modeling of complex baryonic processes within galaxy clusters, or if they serve as indications of new physics beyond the ΛCDM model. Rigorous studies are required to test the viability of alternative models and determine their ability to explain the observed behavior of galaxy clusters. [28]

B. Comparison with other cosmological probes

In this subsection, we compare the results of studying galaxy clusters with other space probes to gain a more complete understanding of the universe and its fundamental dynamics. Additionally, we study the link between the properties of galaxy clusters and measurements of cosmic microwave background anisotropy, which provide insight into the early universe and the formation of oscillations primitive. By analyzing these different space probes, we can validate the ΛCDM model from many different perspectives and gain a more complete understanding of the structure and evolution of the universe.

1) Supernova observations and cosmic acceleration:

The literature review findings regarding the relation between galaxy clusters and the ΛCDM model align with the observations of supernovae and the phenomenon of cosmic acceleration. The discovery of the accelerated expansion of the universe, inferred from measurements of Type Ia supernovae, provides significant support for the predictions of the ΛCDM model. [29] defines a new test for the cosmic distance duality relation (CDDR), utilizing data from 61 Type Ia supernovae luminosity distances and measurements of 61 galaxy clusters obtained from the Planck mission and deep XMM-Newton X-ray data. Furthermore, the literature review emphasizes the complementary nature of galaxy cluster studies and supernova observations in constraining the properties of dark energy and providing additional evidence for cosmic acceleration. These combined studies offer a more comprehensive understanding of our universe, enabling us to probe its structure and evolution with greater precision. The consistency observed between these diverse observations strengthens our confidence in the ΛCDM model as a valid description of our universe. However, it also underscores the need for ongoing research and exploration. [29]

2) Cosmic microwave background anisotropy and primordial fluctuations:

The comparisons between the properties of galaxy clusters and the measurements of cosmic microwave background (CMB) anisotropies provide robust support for the theoretical framework of the ΛCDM model. [30] offers a comprehensive analysis of this comparison. The researchers in this study employed multi-wavelength data from various surveys and observatories to study the properties of galaxy clusters, CMB anisotropies, and galaxy clustering. The study combined galaxy cluster data from the ROSAT All-Sky Survey and the Chandra X-ray Observatory, CMB data from the Wilkinson Microwave Anisotropy Probe (WMAP), and galaxy clustering data from the WiggleZ Dark Energy Survey, the 6-degree Field Galaxy Survey, and the Sloan Digital Sky Survey III. This extensive dataset allowed for a detailed investigation of the properties of galaxy clusters and their relation to the CMB and large-scale galaxy distribution. The literature review confirms that the properties of galaxy clusters, such as their abundance and clustering, align with the constraints derived from CMB observations. The concordance between the observed properties of galaxy clusters and the CMB measurements suggests a common origin for the large-scale structures observed in the CMB and the distribution of galaxy clusters. These findings provide valuable insights into fundamental processes during the early universe, including inflation and the generation of primordial fluctuations. The study also tested the consistency between the cosmic growth of structure predicted by General Relativity (GR) and the cosmic expansion history predicted by the ΛCDM model. [30]

3) Weak gravitational lensing and large-scale structure:

The analysis of weak gravitational lensing and large-scale structure in the literature review serves to provide robust support for the predictions of the ΛCDM model. [31] present a comprehensive examination of the theory and observational status of cosmic shear, a technique that involves measuring the weak distortions induced by gravitational lensing on the shapes of distant background galaxies as their photons traverse large-scale structures. Cosmic shear is used to probe the mass distribution of galaxy clusters. The observed lensing signals from galaxy clusters align with the theoretical expectations based on the distribution of dark matter, thus confirming the role of dark matter in shaping the large-scale structure of the universe. Moreover, the analysis of weak gravitational lensing and large-scale structure strengthens our understanding of the intricate connection between the observed mass distribution of galaxy clusters and the underlying cosmological framework. [31]

VII. CONCLUSION

The ΛCDM model and galaxy clusters are key to understanding the universe's structure and evolution. The ΛCDM model, which includes dark matter and dark energy, helps reconcile theoretical predictions with observational evidence. Cosmological parameters derived from this model reveal the universe's properties, including its matter density, dark energy density, and curvature. Galaxy clusters provide insights into the formation and evolution of cosmic structures. Observational methods like optical observations, X-ray observations, and gravitational lensing techniques offer valuable data on galaxy clusters. However, challenges such as biases, measurement uncertainties, and contamination need to be addressed. The study of galaxy clusters can help determine cosmological parameters like the matter density parameter (Ωm), dark energy equation of state (w), and the Hubble constant (H0). The constraints on cosmological parameters obtained from galaxy clusters can be compared with other observational probes, such as observing supernovae, cosmic microwave background anisotropy, and weak gravitational lensing. The study of galaxy clusters and their significance for cosmology opens exciting prospects for future research. Future research must include improvements in the observational methods and techniques, advancing these tools will allow inferred data with low errors. These improvements could include the development of new high-resolution imaging, spectroscopic techniques, and multi-wavelength observations that provide insights into the interactions between baryonic matter and dark matter within galaxy clusters. A more complete understanding of baryonic physics will lead to more accurate predictions. This synergy will allow for cross-validation of results and tighter constraints on cosmological parameters. Galaxy clusters can also be used to test alternative cosmological models and exotic physics scenarios, providing insights into the validity of the ΛCDM model.

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