Predicting The Eventual Evolution of Neutron Stars into Blackholes Using a Supervised Machine Learning Model.

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Abstract

Although neutron stars are one of the most fascinating objects in the universe, they each have their own special characteristics, such as their magnetic field, deformation rate, and rotating speed. The lifetime of neutron stars that end up in black holes is facing insufficient focus in research and exploration. There is not enough data on this phenomenon, whereas studying this phenomenon opens doors in space exploration and research in this field. The objective of this research is to help scientists, astronomers, and space agencies be able to classify and detect these phenomena in space, improve their understanding of them, and make their space journeys more efficient and easier. This research will make a huge contribution to the understanding of this phenomenon, in addition to collecting more data on it that could turn space exploration into its highest state. A proposed supervised machine learning model (ML) was implemented based on a decision tree to solve multiple equations derived from popular theories, such as the equation of state of neutron-rich and dense matter and the Tolman-Oppenheimer-Volkoff limit. To classify the evolution of neutron stars in binary systems, whether they are about to collapse into black holes or not. Decisions would be based on the mass of the neutron star, and since the data gathered had fewer than clear results, the accuracy of the model was 100%, knowing that when having more sufficient data and following the methodology presented here, the accuracy would differ.

I. Introduction

The purpose of studying neutron stars is to understand the behavioral mechanisms of matter and the nature and structure of both the universe and gravity [1]. In 1939, the first calculation of neutron star models was proposed by Oppenheimer and Volkoff. They assumed that matter is composed of dense, free neutron gas [2]. Ultimately, a supermassive star's core collapses, causing protons and electrons to scrunch together, leaving behind what's called a neutron star. Neutron stars pack approximately between 1.3 and 2.5 Mâ˜‰; however, their surface is city-sized, perhaps 20 kilometers across. A sugar cube there would weigh over 1 billion tons since the matter is cracked so tightly [3].

The acceleration due to gravity at the surface of a neutron star is more than 10^{11} times compared to the surface of Earth which makes them strongly magnetized, and the most rapidly rotating bodies in the Galaxy [4]. Neutron stars' nuclei cannot exist at densities exceeding $\sim (1.5 - 2.0) \times 10^{14}$ g (about the weight of a liter of water) cm^{-3}. At these densities, the matter becomes a uniform plasma of neutrons, protons, and electrons [5]. Black hole study offers the opportunity to investigate the evolution of galaxies, especially dwarf galaxies. It gives a hint about the galaxies that were present closer to the Big Bang [6]. Black holes are a dense matter that even light cannot escape its event horizons because of their huge gravity. Their masses range from supermassive black holes, whose masses range from 100,000 to billions of times the mass of the Sun to stellar black holes, which are born from the passing of stars that is more massive than the Sun [7]. Escape velocity, the speed needed by an object to break free of a planet's gravity can be increased either by increasing the mass within a given radius or decreasing the radius without changing the mass. When escape velocity > C (Speed of Light), even light can't escape, and the enclosed region becomes a hole [8]. **Einstein's theory of gravity** states that when the radius of matter at a fixed mass M is reduced to a small region with a size known as **Schwarzschild radius**, Rs(M), the object becomes a black hole and anything that passes inside that radius can't escape even light [9]. However, Pulsar stars both gain mass and decrease in radii. If the mass of the neutron star is increased by the fallback of material that was located outside the collapsing degenerated core of the exploding star whether in binary systems or when it merges with another neutron star, the neutron stars would occupy a range in mass. If its mass exceeds the mass allowed by the **Equation of state (EoS)**, known as the **Tolman-Oppenheimer-Volkoff (TOV) limit**, the force of gravity overwhelms all other forces turning into a black hole creating a mannerism at the center [10]. It is likely that the maximum neutron-star mass is determined by the stiffness of the EoS and is expected to be about 5 MA. Under the aim of exploring the miracles of both neutron stars and black holes, a decision tree machine learning (ML) model was reached out to serve as a network to connect us with the world of universe evolution. Its target is to classify pulsar stars according to whether they would end up in black holes or not, depending on their masses, Schwarzschild radii, gravitational waves, density, and pressure.

II. Literature Review

i. Formation of Black Holes

According to NASA space discoveries, massive black holes could form when a supermassive star collapses. Relatively small black holes might form if two neutron stars merge, producing ripples called gravitational waves [11].

ii. The Fate of Neutron Stars

Despite the enormous density and heat of neutron stars. During this time, neutron stars cool and become dark, cold and lifeless. Conversely, when neutron stars in binary star systems accumulate enough mass to exceed the Tolman-Oppenheimer-Volkoff limit, they instead form black holes in an event known as a neutron star black hole merger. This event is fundamental to understanding the properties of neutron stars and the evolution of black holes. There are several ways a neutron star can accumulate a mass of. A process called mass transfer can occur, in which a nearby star transfers mass to another star through gravitational interaction. Another common way is the collapse of two stars. In this case, two neutron stars could collapse and instantly form a black hole.

Another common way is the collapsing between two stars, here the two neutron stars may collapse to immediately form a black hole [12].

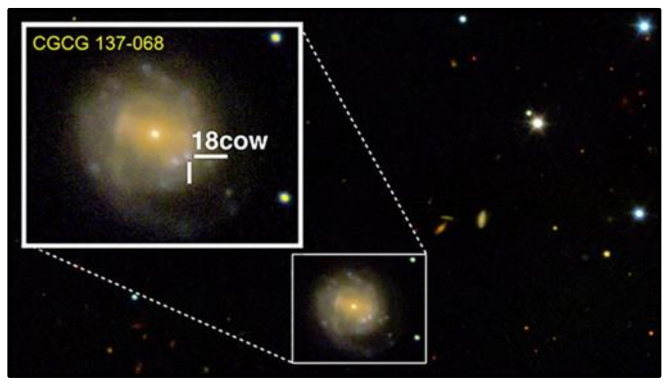

Figure 1 An image of AT2018cow and its galaxy. The first obtained neutron star to turn into a black hole.

iii. The History of Neutron Stars

In the late 18th century, John Michell and Pierre Simon Laplace independently hypothesized that a star could be so massive that not even light could escape its surface. However, research into these phenomena did not continue until the 20th century, when Albert Einstein published his general theory of relativity. The theory described the influence of mass on the structure of space-time. In 1916, Karl Schwarzschild calculated the general relativity equation in the extreme case and discovered that the structure of space-time would collapse on itself, creating a region of zero volume and infinite density from which neither matter nor light could escape black holes [13].

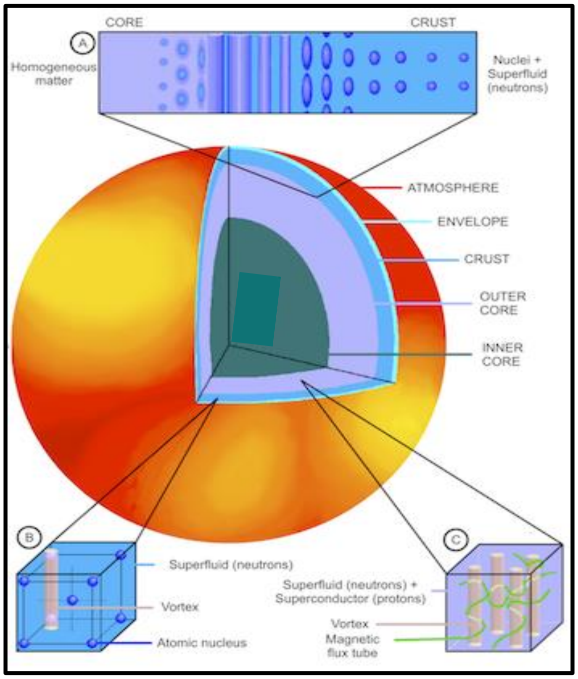

Figure 2 Structure of neutron stars as imaged by Chandra X-ray Observatory.

iv. Detecting Neutron Stars

The Gamma-ray Large Area Space Telescope (GLAST) will allow astronomers to detect even the most energetic and youngest pulsars in the Milky Way galaxy and study the acceleration of particles in space [14]. Their energy is extremely high, varying between 20 to 300 GeV [15].

v. Discovering Black Holes

The existence of black holes was postulated in the 1960s. The complete assurance of their discovery was in the late 1990s in the Milky Way and other nearby galaxies. Since that time, huge theoretical and observational research has been made to understand the astrophysics of black holes [16].

1. Signs of Discovering Black Holes

In the past, the most significant signs of black holes were from a few bright and relativistically beamed sources. However, now, one of the clearest signs of black hole activity is the presence of a compact radio core in the nuclei of a galaxy. New surveys show that nearly all black holes generate compact radio emissions that could be hugely significant in large radio surveys. These cores can have enormous significance in studying the emergence of black holes and detecting the first generations of supermassive black holes [17].

vi. Usage of Machine Learning in Astrophysics

Machine learning models in astrophysics are often used on manually labeled data to automate tedious tasks on large survey datasets. However, there is a trained model on real simulations identify galactic globular clusters (GCs) that have black hoes within it. The goal was to help observers search for stellar-mass black holes and to better understand the dynamical history of black hole clusters by identifying and studying them individually. The Model was selected for 18 GCs transparent enough to accommodate a black hole subsystem. The clusters designated by the ML classifier include M10, M22 and NGC 3201 [18].

vii. Gravitational Waves Detectors

There are several gravitational wave detectors, including the Laser Interferometric Gravitational Wave Observatory (LIGO) in the USA and Virgo and GEO in Europe. Although these devices' success is not fully guaranteed, soon the global network of gravitational wave detectors will be sophisticated enough to record many signals of astronomical origin, large enough to allow the analysis of waveforms that can reveal the structure of the sources, and sufficiently extensive and superfluous enough to show the location of gravitational waves. Sources in the sky through triangulation with this functional network it should be possible to verify the fundamental physics of gravitational waves as predicted by general relativity. Although advanced

LIGO is expected to enable key detection and astrophysics in most categories of gravitational waves [19].

viii. The Usage of Interferometers

The absolute purpose of the Laser Interferometric Gravitational Wave Observatory (LIGO) is to allow the detection of astrophysical gravitational waves and use this information for research in physics and astronomy. In this research, the main goal of studying LIGO devices is to support research on the structure and equation of the state of black holes. In addition, it measures the birth rates, masses, collisions, and distribution of black holes and neutron stars [20].

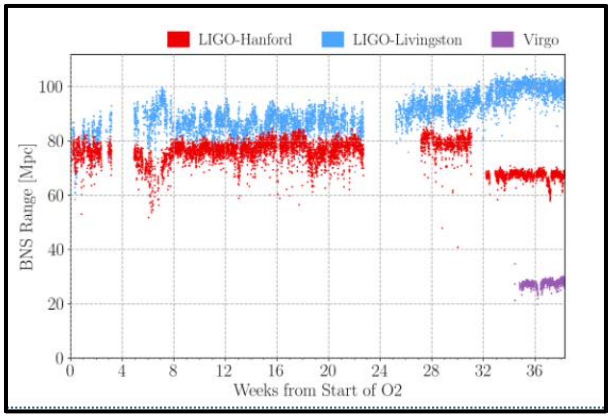

Figure 3 First GW received from a Binary-Neutron-Star merger in 2017 called 'GW170817' ranging from 60 to 80 Mpc (Million Perceps) [21].

ix. Latest Updates

The third round of LIGO observations (O3) revealed several candidates for a neutron star-black hole merger (NSBH) [22]. Studying a single gravitational wave (sin-sin) allows us to see different types of star clusters: young clusters, globular clusters, and galactic nuclei [23]. On September 14, 2015, at 09:50:45 UTC, the LIGO Hanford Observatory in Washington state recorded the convergent signal GW150914 shown in the figure 4.

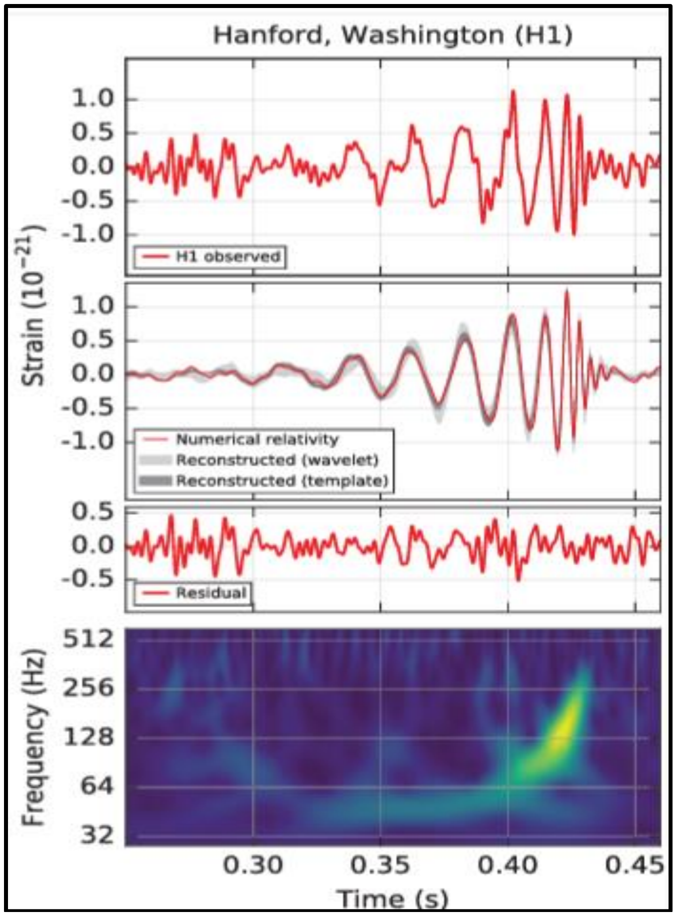

Figure 4 GW150914 is produced by the coalescence of two black holes from their orbital inspiral and merger to the final black hole ring down. Over 0.2s, the signal increases in frequency and amplitude in about 8 cycles from 35 to 150Hz, where the amplitude reach.

x. Measurement Methods of Neutron Stars' masses

Radio observations of rotating pulsars have been shown to provide the most sensitive and precise measurements of neutron star mass [25]. Pulsar timing is one of the sensitive techniques that provides accurate and precise masses of astrometric, rational, and orbital parameters of pulsar stars by adjusting the observed pulse arrival times (TOA) based on the Solar System ephemeris to convert the TOA measured at the l observatory in Center of the cylinder of the solar system. The masses measured using this method are consistent with the best-known masses measured from spacecraft observations, which are likely to provide the most accurate measurements [26].

III. Methods

The machine learning model was based on the **Decision tree** algorithm, which has the function of solving a sequence of equations utilizing inputs from the dataset, such as neutron star masses, Schwarzschild radius, actual radius, density, gravitational waves, and pressure. Multiple datasets would be gathered and combined to match the needed parameters for each equation. The model will be developed in such a way that it could first measure each star's Schwarzschild radii and compare them to the actual radii to determine which ones were most likely to collapse into a black hole This was regarded as the **first selection** method to reduce the number of neutron stars believed to be on the verge of collapsing into a black hole.

This would be achieved by providing the model with:

$$Star = \begin{cases} Black\ hole, & if\ R_{SC} = R_B \\ Neutron\ star, & if\ R_{SC} < R_B \end{cases}$$

where R_{Sc} is the Schwarzschild radius and R_B is the actual radius. Sense we have the condition that states that the star is black hole if the Schwarzschild radius is greater than or equal to its actual radius and it is a neutron star if the Schwarzschild radius is less than its actual radius. The second selection is to measure the critical mass, at which point the neutron star would collapse into a black hole if any further mass was added, as defined by the equation of state (EoS) and the Tolman-Oppenheimer Volkoff limit.

The **final step** in the selection process was to measure the gravitational waves emitted by each star, with the highest values indicating the presence of gravitational collapse caused by a neutron star-neutron star merger or a neutron star-black hole merger, which refers to the conversion of the pulsar star into a black hole. The model was trained and evaluated using an 80%:20% split. Finally, the accuracy was determined by importing the Sklearn library. It's important to mention that these are the absolute clear, right steps to construct the model; however, due to the insufficient datasets found and the roadblocks faced, the model couldn't follow these steps, and the actual methods were much simpler.

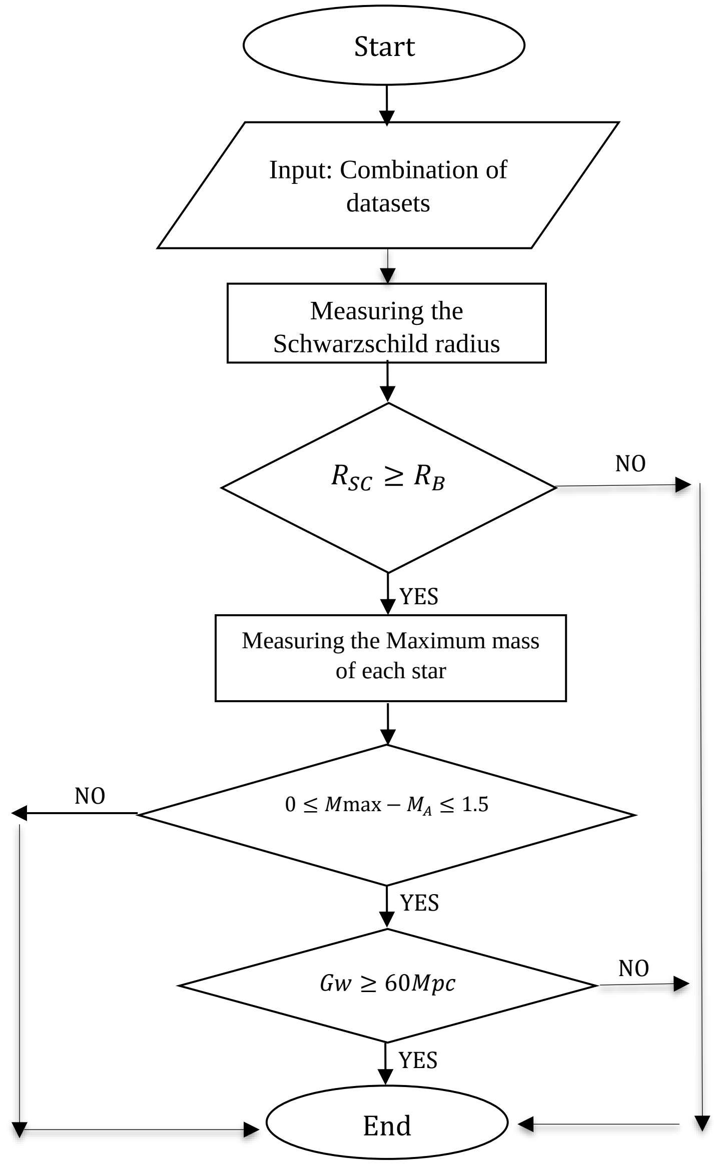

Figure 5 shows a schematic diagram of the model's functions.

The proposed machine learning model was based on the decision tree algorithm, which has the function of classifying and deciding which stars will turn into black holes and which will not, based on multiple datasets that were gathered in order to make clear decisions. Data-cleaning was the first step done in the practical work, removing empty rows and incomplete data to run the decision process smoothly and have fewer errors while coding. According to the fact that the TOV limit was unsuitable for the datasets we gathered, another mass limit was needed to be dealt with in the model; however, it is important to mention that both the TOV limit and the used mass limit here are correct in the sense that the new estimations of the neutron stars' mass limit are starting from the TOV limit. The mass limit that is used here is 2.17 [27]. most of the updated estimations are about this limit. The way the model was trained was by measuring the difference between the star's actual mass and the maximum mass, then building and training the "if" condition that the model would base its decision on. If the mass exceeds the difference, the model will make a 'YES' decision, which means that the neutron star will end up as a black hole. On the other hand, if the mass doesn't exceed the limit, the model will make a 'NO' decision.

IV. Results

The model was trained in a way that it could predict the 'Target' column of the dataset by splitting the data into 80%: 20%. This resulted in 100% accuracy, precision, recall, and F1-score values. This was later interpreted as a result of the few samples contained by the dataset, the well-cleaned data where it was well-separated and easily distinguishable by the features used in the model and lastly the model overfit the training data where it memorized the samples instead of learning general patterns. A Hierarchical cluster graph was implemented by the model through visualizing the data of the two columns 'PSR name' and *'Mp (M* $\odot$)'.

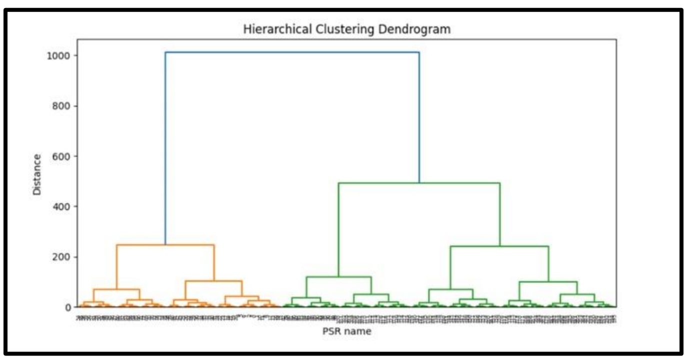

Figure 6 Hierarchical Clustering Dendogram illustrating the data of the pulsar stars names and their coresponding masses.

Another scatter plot diagram was graphed. It visualizes the data of the two columns, the 'Target' and the ''*Mp (M* $\odot$)' column.

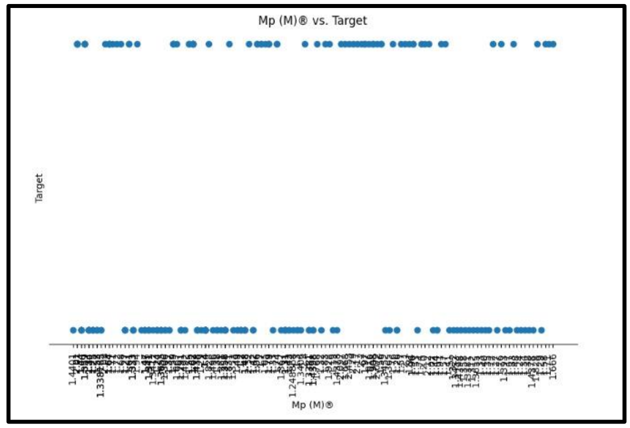

Figure 7 Scatter plot diagram visualizing the data of the 'Target' and 'Mp (M $\odot$*)' columns.*

xi. V. Discussion

xii. Contribution to Space discovery

Our proposed machine-learning model facilitates the observation and understanding of the neutron stars' lifetime. It offers the opportunity to dive deeper into the Big Bang theory and the formation of the universe, additionally, it's a complement to the science of nuclear physics and the nature of matter. Within a few seconds, data collection, analysis, and decision-making would be accomplished whereas solving multiple equations would require days.

xiii. Scientific theories and equations

Many theories upon which our hypothesis was made include **Einstein's theory of general relativity**, T**he Tolman-Oppenheimer-Volkoff limit**, **The Schwarzschild radius** determined by Einstein's theory of gravity, the **Equation of State, and the gravitational waves**:

1. **Einstein's theory of general relativity:** The maximum mass of a neutron star's equilibrium configuration as hypothesized by Einstein's theory of general relativity, Le Chatelier's principle, and the principle of causality wouldn't exceed 3.2 M (solar mass). When the equation of state of matter is unknown in a small range of densities, the extremal principle presented here also applies [28]. The general relativity theory contributed to the idea of the capability of a particle to be converted into a black hole if its energy density exceeded a specific

threshold which was considered as a parameter for the neutron star conversion into a black hole.

The general relativity theory only describes the properties of time and space and mechanical and electromagnetic phenomena in the presence of the gravitational field [29]. Since it deals with gravity as a geometric thing that is derived from the curvature of space-time [30].

Space-time is the combination of space and time. Physical space was assumed by Einstein to be a three-dimensional flat spectrum tat is also an adjustment of all conceivable point locations, whereas time was postulated as a one-dimensional, independent spectrum from space [31]. The spacetime coordinates are written in superscripts as x^k where k = 1, 2, 3, 4. Here the points x1, x2, and x3 describe the spatial coordinates, and x4 describes the independent time coordinate [32]. Einstein field equations are the very fundamental equations of general relativity [33]. These equations are nonlinear, partial deferential, and have four independent variables [34]. Solving such equations can contribute allowing us to reach answers to various physical phenomena in space such as black holes, the death and birth of stars, besides the dynamics of planets. The general relativity theory was one of the theories accepted in the science community after passing all the tests needed for its verification by observation of gravitational lenses [35].

For Einstein to derive the field equations for gravity, he made two interrogated steps. First, he obtained those field equations in a vacuum. Second, he obtained these equations while matter was present from those equations in vacuum. From here, Einstein generated his brilliant theory that he used geometry as a miniature of gravity. Having a simple overview, he used vectors, differential geometric background connected to curvilinear coordinates, tensors such as the Ricci tensor, the Riemann curvature tensor, Christoffel symbols, and the metric tensor [36]. According to general relativity, the curve of spacetime is linked to its matter content. That is the reason the field equations are divided into three parts: the explanation of curvature, the explanation of matter, and the relation between them. The explanation of curvature is specified with respect to the Riemann curve tensor. However, in the two-dimensional case, the Gaussian curvature is what specifies the intrinsic curvature. The explanations of matter include the energy density, the momentum density, and the stress. Thus, these add up to a total of ten quantities. The field equations relate the energy-momentum-stress tensor and the Riemann curvature tensor. They are a set of ten equations that correspond to the ten quantities in the explanation of matter. Using a specific choice of coordinates shows the quintessence of this relation by considering a point in spacetime and a local inertial system in the environs of that point [37].

$$R_{\mu v} - \frac{1}{2} Rg\mu v = \frac{8\pi G}{C^4} T\mu v \quad (1)$$

From this equation, the Einstein field equation that consists of a complex system of ten equations was derived.

2. Tolman-Oppenheimer-Volkoff (TOV) limit:

Oppenheimer & Volkoff were the first to prove that the neutron stars are stable, their masses become maximum for the stiffest EoS that is consistent with fundamental physical constraints [38]. The TOV equation calculates the limit of neutron stars over which neutron degeneracy fails to stop further neutron star collapse to become a black hole-like star.

The mathematical formulation of the TOV:

$$-\frac{dP}{dr} = \frac{G\left[M(r) + 4\pi r^3 \frac{P(r)}{C^2}\right]}{r^2\left[1 - \frac{2GM(r)}{rC^2}\right]} \quad (2)$$

The equation is written in Newtonian mechanics:

$$-\frac{dP}{dr} = \frac{GM(r)}{r^2} \rho(r) \quad (3)$$

where ρ is mass-energy density, M is the gravitational mass of the star, P is the pressure constant and G is the gravitational constant [39].

The Tolman-Oppenheimer-Volkoff limit is simple an integration between the general relativity theory and the general idea of Newtonian gravity. Solving the Tolamn-Oppenheimer equation is built upon solving the EoS of the cold, neutron-rich substance in chemical equilibrium which is the only input needed. Oppenheimer and Volkoff performed the first calculation of the construction of neutron stars by fully depending on the Einstein theory of general relativity. They presented the idea that neutron stars, aided by quantum mechanical compression from their neutrons, will turn into black holes when their mass reaches more than seven tenths of a solar mass. The EoS that is required as an input in order to solve the Tolman-Oppenheimer-Volkoff equation has a relationship between two intensive thermodynamic quantities and the pressure. The most popular EoS is the ideal gas equation:

$P = n\, k_B\, T$. However, this equation will not work for neutron stars because they are objects with very high densities and low temperatures.

This equation explains the natural scale for the energy density in terms of the mass of the fermion and fundamental constants.

The EoS of a free fermi gas of neutrons while the temperature is zero is given by:

$$\varepsilon_n = \frac{(mC^2)^4}{(hc)^3}\frac{1}{\pi^2}\int_0^{x_F} x^2\sqrt{1+x^2}\,dx$$

$$= \varepsilon_0\left[x_F y_{F(x_F^2+y_F^2)-\ln(x_f+y_F)}\right] \quad (4)$$

$$nmc^2\begin{cases}1+\frac{3}{10}x_F^2 & if_{x_F\ll 1}\\ \frac{3}{4}x_F & if x_F \gg 1\end{cases}$$

Here the dimensionless fermi momentum and the fermi energy are given as:

$$x_F = \frac{p_F c}{mc^2} \qquad and \qquad y_F = \frac{\varepsilon_F}{mc^2} = \sqrt{1+x_F^2} \quad (5)$$

As the former relates to the number density as presented

$$n = \frac{k_F^3}{3\pi^2} \qquad PF = hK_F \quad (6)$$

As indicated, the above expression is the exact one. However, it's instructive to provide its leading behavior in both nonrelativistic and ultra-relativistic limits.

$$\varepsilon_0 = \frac{1}{8\pi^2}\,\frac{(mc^2)^4}{(hc)^4} \quad (7)$$

In the zero-temperature limit of interest, the pressure is obtained from the derivative of the energy density with respect to the density. As presented:

$$P = n\frac{\partial\varepsilon}{\partial n} - \varepsilon \quad or \quad P+\varepsilon = n\varepsilon F = \frac{k_F^3 \varepsilon F}{3\pi^2} \quad (8)$$

After all the derivatives, the EoS for energy density is:

$$p(n) = \varepsilon_0\left[\frac{2}{3}x_F^3 \;\; y_F - x_F \;\; y_F + \ln(x_F+y_F)\right]$$

$$nmC^2,\begin{cases}\frac{x_F^2}{5} & if x_F \ll 1\\ \frac{x_y}{4} & if x_F \gg 1\end{cases} \quad (9)$$

As indicated before, the equation of state for neutron-rich, dense matter is all that is required to solve the TOV equation. For now, to study the structure and composition of neutron stars. The TOV equation is presented as:

$$\frac{dP(r)}{dr} = -\frac{G}{C^2}\frac{(\varepsilon_r+P_r)\left(M_r+4\pi r^3\frac{p_r}{c^2}\right)}{r^2(1-2GM_r/c^2 r)} \quad (10)$$

$$\frac{dM(r)}{dr} = 4\pi r^2\,\frac{\varepsilon(r)}{c^2} \quad (11)$$

Here, $M(r)$, E(r), and $P(r)$ are the respective profiles for mass, energy density, and pressure.

All these calculations, which were carried out by Oppenheimer and Volkoff, predict that a neutron star with a value exceeding $M = 0.71\ M_\odot$ will become a black hole. This shows that neutron stars that exceed a certain mass limit will collapse under their weight, and they might attain this mass at a finite radius of $R \approx 9.2$ km in the state of degenerate fermi gas of neutrons.

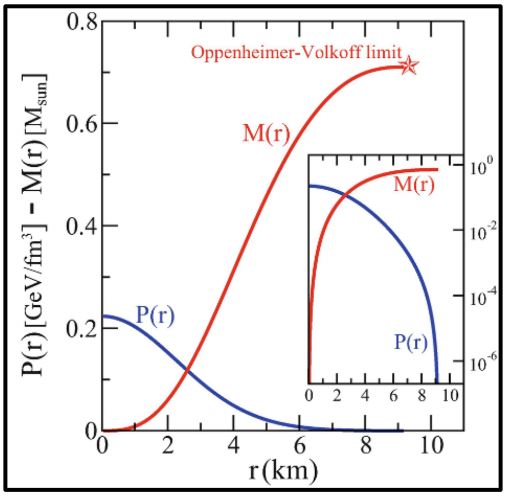

Figure 6 As predicted by Oppenheimer and Volkoff, the Pressure-mass relationship for the heaviest neutron star can be maintained by a degenerate fermi gas of neutrons Volkoff for the first time.

Even though the study of the TOV has been repeatedly used to date, there is now some evidence that suggests that the Fermi gas equation of state is

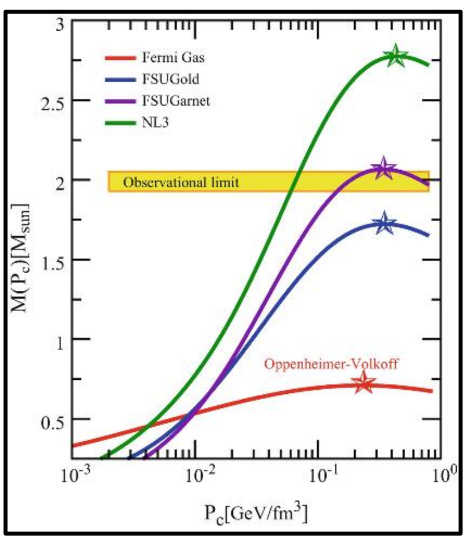

Figure 9 stars masses as with respect to the central pressure as predicted by Oppenheimer and Volkoff. There are three equations of state. Here, the horizontal crew explains the constraints from the mass measurements reported.

not acceptable. Surely, two neutron stars with masses of two solar masses have been recorded. They are nearly three times larger than the limit. This fact shows how critical nuclear interactions are to the structure of neutron stars [40].

3. Schwarzschild radius: It's the radius of the event horizon enclosing non-rotating black holes. If the Neutron star has a mass larger than the solar mass, its radius maybe 1.5-2 greater than the the Schwarzschild radius [41]. As derived by Karl Schwarzschild, any object with a radius smaller than its Schwarzschild radius (R_s) will turn into a black hole.

The Schwarzschild radius equation is presented as:

$$R_{Sch} = \frac{2GM_{eff}}{C^2} \quad (12)$$

Here, G is the gravitational constant, M is the mass and c is the speed of light.

The Schwarzschild radius is derived by using the mathematical relation in the gravitational law [42].

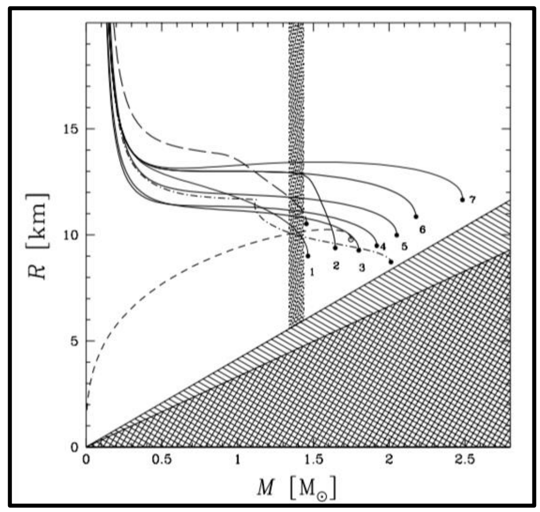

Figure 10 shows the relation between the radius and the gravitational mass, M, for seven Eos of Baryonic matter, labeled by numbers from 1-7.

Escape velocity, the velocity needed to leave the boundary of an object exceeds the velocity of light if the Radius of this mass<R_s [43].

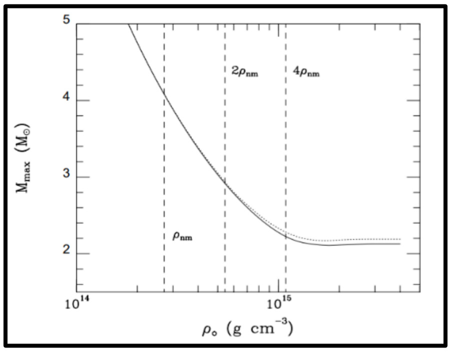

Figure 11 shows the maximum neutron star mass, M_{max}, as a function of the fiducial density, where $\rho_0 = \rho_{nm}$, where nuclear matter density $\rho_{nm} = 2.7 \times 10^{14} g\ cm - 3$

4. Equation of State (EoS): To calculate the maximum mass of a neutron star to be stable against the gravitational collapsing forces using the EoS, following a set of conditions governed by Rhoades & Ruffini (1974). They are imposed on the EoS of the inner core, and they include:

(i) The mass density is non-negative, i.e., gravity is attractive.

(ii)Neutron matter should be fluid where the pressure at zero temperature is a function of density.

(iii) $d\ /\ d\ \rho \geq 0,$ so that the zero-frequency sound speed of neutron matter $(dp\ /\ d\rho)^{1/2}$ is real and matter is stable.

(iv) The sound speed does not exceed the speed of light, $dp\ /\ d\rho \leq c^2$, hence signals cannot be superluminal, and causality is satisfied.

The speed of sound is equal to the speed of light c^2.

For densities below $\boldsymbol{\rho_0}$: To measure the maximum mass of a neutron star at densities below ρ_0, the recent equations of state presented by **WFF88**. The results of WFF88 represent the best microscopic EoS for dense matter constrained by nucleon-nucleon scattering data.

The lowest value of M_{max} is determined by the maximum value of ρ_0 up to which one can be confident of the WFF88 equation of state. Their results can be regarded as valid up to $\rho_0 = 2p_{nm}$ at which density M_{max} = 2.9 M.

For densities lower than $\boldsymbol{2.5 \times 10^{14} g\ cm^{-3}}$: To measure the maximum densities below this density, the equation of state given by Baym, Pethick, & Sutherland (1971) would be used, since the details of the equation of state at these low densities do not affect the results, whereas higher densities than $1.6 \times 10^{15} gcm^{-3}$, the speed of sound exceeds the speed of light [44].

5. Gravitational Waves: Gravitational waves (GWs) are ripples that travel as waves to other places in the universe. The mass of an object to be identified as a black hole is not enough to only be estimated, but also the gravitational waves of the neutron star for stability against collapse into a black hole should be taken into consideration [45]. Most of the neutron stars, after accreting huge masses enough for the conversion turn into a type of black hole classified as a “Schwarzschild Black Hole”. They have neither charge nor rotation with a Singularity in the middle, also called a "static black hole" or "ideal black hole [46].”

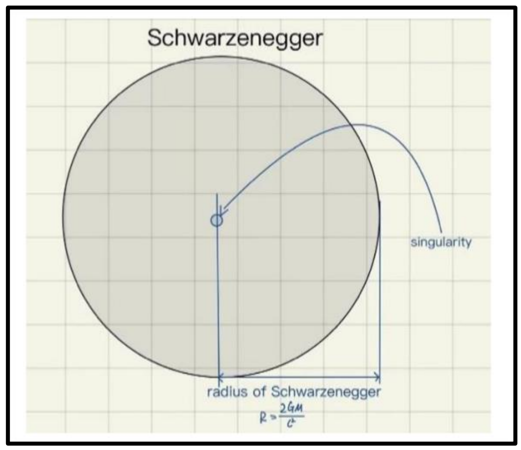

Figure 12 The Schwarzschild Blach Holes

At the Schwarzschild radius, if more mass, greater than M, was accreted, the gravitational waves will oppose and overwhelm any other force converting the neutron star into a dense mass of matter which is the black hole.

The initiation of gravitational radiation from neutron stars is due to the merger of two neutron stars known as “Binary-Neutron star- Mergers” (BNS). During the rotation and merger, the outer surface of both stars come int contact, forming a shear surface and a Kelvin-Helmholtz instability develops, then forms a series of vortices. The cores then coalesce. The merged part ends up in a stationary configuration which further has two fates: a neutron star or after collapsing, a **Kerr black hole**, another type of black holes. Depending on the initial masses of the BNS and on the EoS, the end of the merger is determined.

If the masses were high and the EoS was soft, the merger would end up collapsing into a black hole, while BNS with lower masses and stiffer EoS wouldn't collapse into a black hole [47].

xiv. Achievements

Easy classifications of neutron stars whether they are near to collapsing into a black hole or not were made by the ML model without the need for solving complicated equations where the same function was achieved by the model with a nearly high accuracy compared to the found datasets. Devices, such as LIGO and Virgo, and models were proposed to be merged and under the control of an IoT platform, they were capable of interchanging real-time data and presenting high processing and analysis capabilities to discover the miracles of Astrophysics and the secrets of Artificial Intelligence.

xv. Limitations and roadblocks

Linking our model to LIGO and Virgo devices to continuously be provided with real-time data wasn't accomplished due to time constraints and not knowing all the aspects of the subject since the main goal was to ensure the validity of our experiment. Due to the lack of sufficient datasets related to the equations' inputs led to a slightly low accuracy and inability to implement a whole prototype performing the functions defined, as a result, a small, limited prototype was implemented to test both the accuracy and the capability of it to perform a similar idea.

xvi. Future Plans

One of the recommended additions is linking the real-time data provider devices to the model to both increase the accuracy by increasing the model training time on various data and be ready for space agencies to use and conduct their research. Furthermore, developing the model to be able to predict the conversion of the neutron star into a black hole in what function of time and the probability of the conversion. The main and most important future plan is to construct a more sufficient and successful model by implementing the methodology recommended in this research, due to the fact that the data provided was not enough.

V. Conclusion

The machine learning model that was proposed here was a supervised one, implemented on the decision tree model, to solve several equations in an untheoretical, much easier way that can help to explain and discover this phenomenon more. The equations that are purposed are based on significant theories such as Einstein's general theory of relativity, the Tolman-Oppenheimer-Volkoff limit, the equation of state for cold, neutron-rich matter, the Schwarzschild radius, and gravitational waves. Machine learning is a remarkable method in astrophysics as it produces much more accurate results than solving equations theoretically, and it can be linked to tools in space. The decision of the model is going to be based on real-time analytics from the Laser Interferometer Gravitational Waves Observatory (LIGO) and its other devices. The parameters used to classify this phenomenon include mainly mass, radius, density, and pressure. As some hardships were faced, the methodology process was not fully clear and the accuracy measured was 100%, it is expected that the accuracy will differ when the clearest methodology is implemented. However, this remarkable method will face more and more improvements later on.

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VII. References

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VIII. Appendix

https://github.com/HannahMohamed66/BNS.git