Detection of Dark Matter in Neutron Stars

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Abstract

Despite the latest advancements in the field of astrophysics, dark matter, the vague mysterious thing occupying vast parts of our universe, is still floating around with its obscure nature and very hard to detect components. This research is delving into different approaches for detecting dark matter present in neutron stars and calculating its mass. The approaches discussed are considered secondary techniques in the detection of dark matter and they are being scrutinized with the aim of developing a new way of detecting these vague particles. For instance, hybrid detection methods like star spectroscopy and Gravitational Red Shift Measurements, direct detection methods like scattering experiments, and indirect detection methods like neutrino emissions. Results have shown that some specific neutron stars have higher expectancy for the presence of dark matter particles and some specific techniques are more feasible than others in their detection. By making the most of the latest technology like high-resolution telescopes and software programs, it is possible to utilize dark matter's effects on neutron stars to assist in its detection. We believe that the detection of dark matter and calculating its mass can pave the way for some revolutionary discoveries regarding the hidden intricacies of the universe.

1. Introduction

After years of research, we are still far from comprehending dark matter's properties and identifying its components. Dark matter's enigmatic nature continues to captivate scientists as it constitutes the major portion of the universe. It cannot be seen directly as it consists of weakly interactive particles that do not interact with any form of electromagnetic radiation making its detection extremely challenging. However, its existence is inferred from its gravitational effects on visible matter.

"Dark matter is thought to be the glue that holds galaxies together" [1] and it has a significant role in the formation of large-scale structures in the universe, such as clusters of galaxies. Hence, the quest to unravel the mysteries of dark matter remains a crucial challenge as it can pave the way for some revolutionary discoveries regarding the hidden intricacies of the universe.

Neutron stars are thought to be potential candidates for dark matter interactions due to their extreme density and strong gravitational field which makes dark matter, although it is weakly interacting, interact with the neutrons in a neutron star through the weak nuclear force.

In this article, we are specifying neutron stars to be utilized for the detection of dark matter and calculating its mass through multiple approaches. We think that Proving interactions between dark matter and neutron stars will be a breakthrough in our understanding of dark matter. Moreover, it will provide new insights into the nature of neutron stars. and the physics of the weak nuclear force.

I. Dark Matter Candidates and Interactions

Dark matter is a hypothetical form of matter that is thought to account for about 27% of the matter in the universe. [2] It is invisible to telescopes because it does not emit or absorb any form of electromagnetic radiation. However, its existence is inferred from its various effects on visible matter.

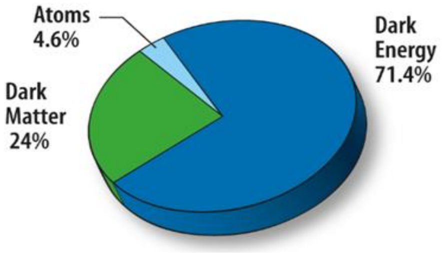

Figure 1 Pie Chart Indicating the distribution of Dark Matter [3]

i. Evidence that supports the existence of dark matter

A. The rotation curves of galaxies: This curve is a plot describing the rotation rate in a galaxy as a function of distance from the galaxy's center. It shows that the stars in the outer parts of galaxies are moving much faster than they would be if they were only affected by the gravity of the visible matter in the galaxy. This suggests that there is a lot of unseen matter in the outer parts of galaxies, which is holding the stars in place.[4]

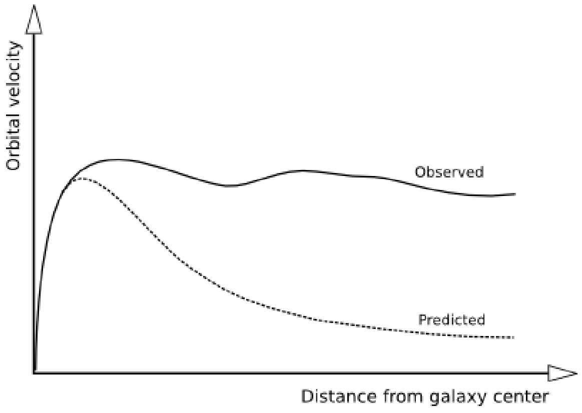

Figure 2: Description of the rotation rate of a galaxy [5]

B. The gravitational lensing of galaxies: Gravitational lensing is the bending of light by the gravity of massive objects. When light from a distant galaxy passes by a nearby galaxy, the gravity of the nearby galaxy bends the light, causing the distant galaxy to appear distorted. This distortion can be used to measure the amount of dark matter in the nearby galaxy. [6]

ii. Dark matter candidates

All the proposed candidates for dark matter are hypothetical particles that have not yet been detected. However, scientists are currently conducting several experiments aimed at detecting these elusive particles.
Weakly interacting massive particles (WIMPs): These are hypothetical particles that interact with ordinary matter through the weak nuclear force. They are thought to be a good candidate for dark matter because they have the right mass and properties to explain the observed gravitational effects of dark matter. WIMPs have the right mass and properties to explain the observed gravitational effects of dark matter. Nevertheless, their existence faces several theoretical challenges.
Axions: Axions are hypothetical light particles that interact weakly with other particles, which makes them a good candidate for dark matter because they would be difficult to detect, yet still able to explain the observed gravitational effects of dark matter.
Sterile neutrinos are another hypothetical particle that is similar to neutrinos, but they do not interact with the weak force. They could potentially explain the observed gravitational effects of dark matter, as well as the observed abundance of light elements in the universe. Nevertheless, their existence is still uncertain, and further research is needed to confirm their existence and properties.

II. Redshift: its relationship with dark matter

Redshift is a phenomenon in which the wavelength of electromagnetic radiation is increased as a result of the relative motion of the light source and observer. This increase in wavelength is accompanied by a decrease in frequency and photon energy. It is called "red shift" because when the wavelength gets elongated it is displaced towards the red light on the electromagnetic spectrum.[7] The cosmological redshift of light depends on both the speed of the emitter and
the distance between the emitter and the observer. In case the emitter is moving away from us, a redshift is observed. If the emitter is moving towards us, whether a redshift, a blueshift, or no shift is observed will depend on the speed vs. the distance. When the speed is in the range of $c(\exp[-\beta D] - 1) < v < 0$, with β being the redshift constant, a redshift is observed; if the speed equals $c(\exp[-\beta D] - 1)$, no shift is observed; if the speed v is less than $c(\exp[-\beta D] - 1)$, a blueshift is observed. [8]

i. Factors causing redshift:

A. Doppler effect

When a light source is moving away from an observer, the light waves are stretched out, making their wavelength longer. This is similar to what happens when a sound source moves away from the listener, the further the sound source gets, the lower the pitch of a sound wave.

B. Gravitational redshift

When light travels through a gravitational field, its wavelength is increased as it has to work against the gravitational field to escape which slows down its speed. The stronger the gravitational field, the more the light is stretched and the redder it becomes.

ii. Dark matter effect on the gravitational redshift of cosmological bodies

Since dark matter is proved to affect the gravitational field, it is believed that the presence of dark matter as halos around neutron stars can cause significant difference in the amount of the star's redshift, which can consequently assist in calculating the amount of dark matter causing this difference.

III. Neutron Stars: Composition and Properties

i. Formation and Composition

Neutron stars are the remains of massive stars that died in what is known as a supernova. The initial star’s mass is between 10 and 25 solar masses. When these stars run out of fuel, they collapse under their own gravity and form neutron stars. Their outer layer is a solid crust formed from normal matter. It can reach up to 10 kilometers in thickness and it is made up of ions and electrons. The internal part is made up of neutrons, electrons, and protons. The neutrons are in the form of gas, packed tightly forming a degenerate fermion gas. Protons and electrons are also degenerate fermion gas but are less dense than neutrons. The core of these stars may contain exotic matter like quark matter and strange matter. However, information about the core is yet to be verified.

ii. Properties

Neutron stars are incredibly dense, with a mass of about 1.4 times that of the sun, but a radius of only about 10 kilometers. Their density is about 10^17 grams per cubic centimeter, which is about 100 million times the density of lead. It is often said that “A tablespoon of a neutron star material would weigh more than 1 billion U.S. tons (900 billion kg). That's more than the weight of Mount Everest, Earth's highest mountain.” [9]

Neutron stars’ temperatures can reach up to 10 million degrees Celsius. This heat is produced by the collapse of the star's core and the friction of the neutrons as they collide with each other. They are also extremely magnetic, with magnetic fields that are billions of times stronger than the Earth's magnetic field. These strong magnetic fields are thought to be generated by the rotation of the star.

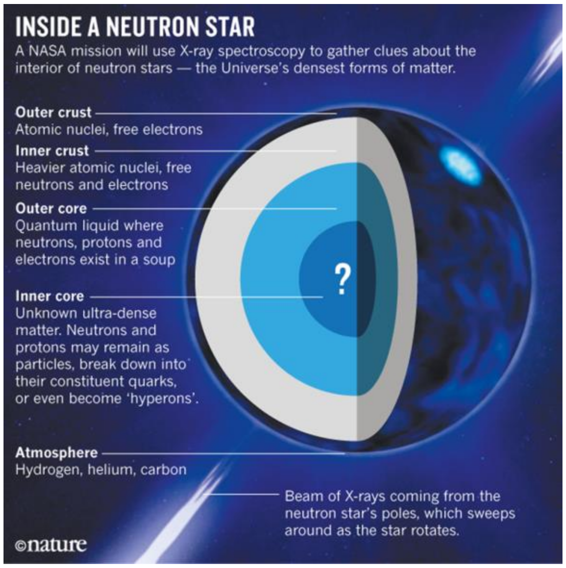

Figure 2: Neutron Star Composition [10]

iii. Types of Neutron stars

Neutron stars have multiple common properties such as the ones mentioned above. However, they can be distributed into groups, each having special properties.

A. *Pulsars* are neutron stars that emit beams of radiation, usually radio waves. The beams are emitted from the poles of the neutron star as the star rotates. This causes the pulsar to appear as if it is blinking as it rotates. “Because of the rotation of the pulsar, the pulses thus appear much as a distant observer sees a lighthouse appear to blink as its beam rotates. The pulses come at the same rate as the rotation of the neutron star, and, thus, appear periodic.” [11]

B. *Magnetars* are an exotic type of neutron stars with extremely strong magnetic fields reaching up to a trillion times stronger than the magnetic field of Earth. Magnetars release enormous amounts of x-rays and gamma-rays making them responsible for some of the most powerful explosions in the universe, such as gamma-ray bursts. [12]. It is believed that “its magnetic field would destroy your body, tearing away electrons from your atoms and converting you into a cloud of monatomic ions, that is, single atoms without electrons.” [13]

C. *X-ray binary Systems* are systems that contain a neutron star and a companion star. The companion star can be a main sequence star, a white dwarf, or another neutron star. The neutron star in an X-ray binary accretes matter from the companion star, which causes it to emit X-rays.

Property	**Magnetars**	**Pulsars**	**X-ray binaries**
Magnetic field	Extremely strong (10^14-10^15 gauss)	Strong (10^8-10^12 gauss)	Variable, depending on the type of binary system
Rotation period	Varies from milliseconds to seconds	Varies from milliseconds to seconds	Variable, depending on the type of binary system
Power source	Magnetic field	Rotational energy	Accretion of material from the companion star
X-ray emission	Yes, persistent and variable	Yes, pulsed	Yes, pulsed
Gamma-ray emission	Yes, during flares	no	sometimes

Table 1: Comparison of types of neutron stars

iv. Advantages of Binary Neutron Star Systems for Research:

The stars in binary systems are the most feasible type of neutron stars for research and study. Firstly, they are brighter as they reflect light off of each other which makes them easier to study with telescopes.
Binary stars are more stable as they are held together by their mutual gravitational attraction. On the other hand, single stars can be affected by the gravitational pull of other stars in their vicinity, which can make it difficult to study them.

v. Mass limit of Neutron stars

This is the maximum mass a neutron star can possess before collapsing under its own gravity into a black hole. This happens because the strong nuclear force is not strong enough to hold the neutrons together against the immense gravity of the star. The limit is called the Tolman–Oppenheimer–Volkoff limit (TOV limit), and it is estimated to be around 2.16 solar masses. It is determined by the equation of state of neutron matter, which is the relationship between the pressure and density of neutron matter. However, the TOV limit is not 100% certain. For instance, PSR J0740+6620 is a neutron star that has a mass of about 2.14 solar masses. This immense mass suggests that the TOV limit may be slightly higher than 2.16 solar masses.

IV. Potential interactions between dark matter and neutron stars

It is scientifically feasible to predict the presence of dark matter in the cores of neutron stars since it does not interact with electromagnetic radiation and can therefore flow into the dense matter of these stars without being detected. Recent observations have revealed that old neutron stars are experiencing a notable increase in temperature to near-infrared ones. This phenomenon has led scientists to speculate that dark matter may be penetrating the dense matter of these stars and causing changes in their properties. This flow of dark matter has detectable effects on the surface of these neutron stars, particularly those located in dark-matter-rich regions like the Galactic center or cores of globular clusters.

i. The density of neutron stars

Neutron stars are known for their incredibly dense nature. This density causes a critical amount of spacetime curvature, leading to a powerful gravitational field. Due to this strong gravitational field, it is more likely for weakly interacting dark matter particles to be captured. Therefore, it is scientifically feasible to predict the presence of dark matter in the cores of neutron stars.

“Because of their strong gravitational field, neutron stars capture weakly interacting dark matter particles (WIMPs) more efficiently compared to other stars, including the white dwarfs. Once captured, the WIMPs sink to the neutron star center and annihilate.” [14]

ii. Scattering off the neutrons in the star.

Dark matter can enter neutron stars by scattering off the neutrons in the star. This is because dark matter is thought to be made up of weakly interacting massive particles (WIMPs), which can interact with neutrons through the weak nuclear force. When a WIMP scatters off a neutron, this would cause the WIMPs to lose energy and eventually be captured by the neutron star. Furthermore, it can transfer some of its momentum to the neutron, causing the neutron to move.

Scattering of neutrons in the star can have several effects that are easily detected, including the heating of the neutron star, the change in its rotation rate, the emission of gravitational waves, and the production of neutrinos.

Ongoing research was conducted with the aim of studying the scattering of neutron stars with dark matter. Firstly, The European Pulsar Timing Array (EPTA), is a network of radio telescopes used to measure the arrival times of radio waves from pulsars. The EPTA studied the timing of these waves so that scientists could look for changes in the rotation rate of pulsars that could be caused by the scattering of dark matter.

Secondly, The Neutron Star Interior Composition Explorer (NICER) is a NASA satellite that is used to study the interior of neutron stars. NICER can measure the temperature and composition of neutron stars, which could help scientists to determine if dark matter is present in these stars.

Thirdly, The Large Synoptic Survey Telescope (LSST). This is a terrestrial telescope in Chile that will be used to survey the entire sky every few nights. This will allow scientists to search for neutron stars that are being heated or slowed down by the scattering of dark matter.

V. Neutron Star: Structure and Equation of State

The neutron star structure equation of state is a critical component in understanding the internal composition and properties of neutron stars and how dark matter affects them. This equation of state describes how the pressure and energy density of matter within a neutron star depend on its density. In essence, it provides insights into how matter behaves under the extreme conditions present within these stellar remnants.

The equation of state for neutron stars is particularly relevant when considering the detection of dark matter within them. Dark matter, an elusive form of matter that doesn't emit light, might accumulate within neutron stars due to its gravitational interactions. This accumulation could influence the star's internal structure, including its density distribution and pressure profile.

If dark matter were to accumulate within neutron stars, it might affect the equation of state by altering the relationship between pressure, energy density, and density itself. This, in turn, could lead to observable consequences in terms of the star's properties, such as its mass-radius relationship or the behavior of emitted radiation.

The NS equation of state (EoS) relates the pressure, P, to other fundamental parameters. With the sole exception of the outermost layers (a few meters thick) of an NS and newly born NSs, the pressure in the strongly degenerate matter is independent of the temperature. Then, the microphysics governing particle interactions across different layers of an NS is encapsulated in one-parameter EoS, $P = P(\rho)$, where P and ρ are pressure and density, respectively. Calculations of the EoS are frequently reported in tabular form in terms of the baryon number density, nb, i.e. $P = P(nb)$, $\rho = \rho(nb)$. The EoS is the key ingredient for NS structure calculations; its precise determination, however, is an open problem in nuclear astrophysics and is limited by our understanding of the behavior of nuclear forces in such extreme conditions [15].

It is also worth mentioning what Bell et al. said "While the EoS of the outer crust is based on experimental data and is rather well established, physics beyond the neutron drip point cannot be replicated in the laboratory, and theoretical models are used instead. Thus, the EoS of the inner crust and the core are calculated in a reliable way using methods of nuclear many-body theory. Nevertheless, even when considering the simplest NS core made of neutrons, protons, electrons and muons, the reliability of this EoS decreases at densities significantly higher than $\rho 0$, primarily due to our lack of knowledge of strong interactions in superdense matter. The only way to constrain these models is through observations".[15], [16].

Furthermore, several EoSs are found in the literature [16]– [20]

In essence, the neutron star equation of state serves as a bridge between theoretical predictions of dark matter accumulation and observable signatures within these extreme environments. The behavior of dark matter within neutron stars could lead to detectable deviations from expected neutron star properties, thereby offering a potential avenue for unveiling the mysterious nature of dark matter.

VI. Detection Methods of Dark Matter in Neutron Stars

Dark matter particle interaction with earth detectors happens once per year making their study challenging. However, dense objects such as neutron stars are posed as good targets to probe and study dark matter due to their high mass density and compactness. In this section, we analyze the different techniques and methods used in the detection of dark matter in neutron stars and their constraints.

i. Indirect Detection Methods

A. Neutrino Emissions

“The detection of these neutrinos will be complementary to the accelerator- and reactor-based experiments that study neutrinos over the same energy range” [21]. As dark matter particles annihilate or decay, high-energy neutrinos could be yielded as byproducts. These elusive neutrinos traverse through many dense astrophysical environments as neutron stars with relative ease, serving as a “smoking gun” illustrating dark matter interactions [15]. However, the standard model of neutrinos as we know it cannot constitute a significant part of dark matter because of its current upper mass limits. This limit implies that the neutrinos would move too freely across the huge distances in space leading to them spreading out and erasing the patterns we see in the density of matter at those large distances. Nevertheless, many extensions of the standard model suggest interesting neutrino states where there are some extra types of neutrinos for each set of particles known as “lepton generations”. These neutrinos don’t interact with the forces known to us and they are usually termed *sterile neutrinos* although they are not sterile in the strict sense as they are a mix of normal, and active neutrinos [22].

B. Gamma Ray Emissions

Gamma rays are Astro particles with some unique properties making them an excellent choice for indirect searches for WIMP dark matter. Gamma rays are characterized by the ability to travel to the observer without deflection, allowing the mapping of the sources of the signal, and the prompt emission carrying important spectral information that can be used to characterize the dark matter particle in the case of detection [23].

In order to detect gamma rays, the photons have to be observed from space due to the earth’s atmosphere's opaqueness to gamma rays. The Fermi Gamma-ray Space Telescope (FGST) launched in June 2008, and formerly known as the Gamma-ray Large Area Space Telescope (GLAST), is a space observatory that is being used in performing gamma-ray observations. Its main instrument is the Large Area Telescope (LAT). The LAT consists of a pair production detector consisting of an array of modules that form the tracker surrounded by an anti-coincidence detector for charged particle identification [24]. These detectors are sensitive to gamma rays ranging from 20 MeV to >300 GeV. The LAT detects and constructs individual gamma-ray and charged-particle events determining the arrival direction of the event. It operates primarily in sky-scanning mode enabling scientists to conduct studies all over the sky. Furthermore, the GAMMA-400 telescope covers a similar energy range to the LAT but with improved angular and energy resolution. According to a study [25] about its capabilities, it measures the “gamma-ray and electron + positron fluxes using the main top-down aperture in the energy range from ~20 MeV to several TeV in a highly elliptic orbit (without shading the telescope by the Earth and outside the radiation belts) continuously for a long time. The instrument will provide fundamentally new data on discrete gamma-ray sources, gamma-ray bursts (GRBs), sources and propagation of Galactic cosmic rays, and signatures of dark matter due to its unique angular and energy resolutions in the wide energy range. The gamma-ray telescope consists of the anticoincidence system (AC), the converter-tracker (C), the time-of-flight system (S1 and S2), the position-sensitive and electromagnetic calorimeters (CC1 and CC2), scintillation detectors (S3 and S4) located above and behind the CC2 calorimeter and lateral detectors (LD) located around the CC2 calorimeter".

ii. Hybrid Detection Methods

A. Star Spectroscopy

According to a 2019 study [26] "neutron star spectroscopy could constitute the best probe for dark matter particles over a wide masses and interactions strength". By studying the heat caused by dark matter-neutron interactions based on the current population of neutron stars scientists were able to draw in the region in which black matter particles could be detected using star spectroscopy in the luminosity vs. age plane. See Fig (3) [26] for an illustrative drawing of the process.

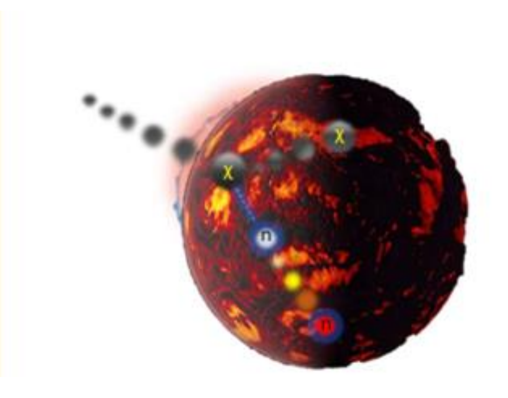

Figure 3: Star Spectroscopy

The reasoning behind this technique relies on the simple conversion from recoil energy to thermal energy [27], in which we compute the energy transferred to a neutron inside a neutron star is the dark matter-nucleon scattering process in a relativist setting. "This energy corresponds to the same recoil energy inferred in direct detection experiments on Earth but considering relativistic effects. For this reason, neutron stars indeed constitute an orthogonal search for dark matter."

The flux of dark matter passing through the neutron star is represented in the following equation:

$$\dot{m} = \pi * b^2 v_x \rho_x \qquad (1)$$

where ρ_x is the dark matter density in the halo, and b is.

$$b_{max} = \left(\frac{2GMR}{V_X^2}\right)^{\frac{1}{2}} \left(1 - \frac{2GM}{R}\right)^{\frac{-1}{2}} \qquad (2)$$

iii. Direct Detection Methods

Direct detection methods of dark matter on Earth rely on the scattering of dark-matter particles from the halo of the Milky Way in a detector on Earth. The latter is usually set up deep underground as "Sanford in South Dakota" [22]. However, when dealing with neutron stars, the process is much different and much more complicated.

A. Scattering Experiments

Scattering is one of the easiest and most used techniques in the detection of dark matter. For dark matter in the GeV mass range (neutron stars), it is possible to impact particles and scatter off of them. This method of detection is limited by the cross-section of the target material used and by the excitation energy of that material. For light-dark matter, elastic collisions can generate nuclear recoils inside of the target's crystal lattice. The energy of the recoil is given by:

$$E_n r = \frac{g^2}{2m_n}\ 1eV * (\frac{m_{DM}}{100MeV})^2(\frac{10GeV}{m_N}) \tag{3}$$

where mN is the mass of the nucleus, q ~ vmDM is the momentum transferred, and v ≈ 10-3c is the DM velocity [28]. As the equation shows, dark matter with a larger mass will produce larger disruptions in the target material’s lattice structure, but larger target masses will reduce the amplitude of the disruption. Scattering is the most common method of detection due to its simplistic setup and higher potential interaction rates.

VII. Constraints and Applicability to Neutron Stars

The distinctive nature of neutron stars engenders both challenges and opportunities for dark matter detection methods:

i. Neutrino Emissions

The dense interiors of neutron stars could augment neutrino emissions from potential dark matter interactions. However, disentangling these emissions from the myriad neutrino sources inherent to neutron stars is an intricate endeavor [15].

ii. Gamma-ray Emissions

The magnetic fields of neutron stars could amplify gamma-ray signals originating from dark matter interactions. Yet, deciphering these signals from the diverse ensemble of gamma-ray sources is a complex puzzle.

iii. Scattering Experiments

While terrestrial scattering experiments are well-established, their adaptation to the extreme conditions within neutron stars remains uncertain [27].

iv. Gravitational Red Shift Measurements

Gravitational redshift measurements offer a tantalizing approach to uncovering dark matter in neutron stars. However, the intricate interplay between various neutron star properties and dark matter interactions presents a challenge [28].
In addition, Table (2) summarizes the problems and advantages of each method discussed.

Particle	**Experiments**	**Advantages**	**Challenges**
Gamma ray photons	Fermi-LAT, GAMMA-400	point back to source spectral signatures	backgrounds, attenuation
Neutrinos	ICE Cube/ Deep core	point back to source spectral signatures	backgrounds, low statistics
Cosmic Rays	PAMELA, CTA, LAT	low backgrounds for antimatter searches	diffusion, do not point back to source

Table 2: Comparison of various detection methods

In conclusion, the quest for detecting dark matter in neutron stars necessitates a multidisciplinary effort involving theoretical insights, observational data, and innovative simulations. The convergence of these approaches will determine the feasibility of detecting and deciphering the enigmatic dark matter within the confines of these stellar remnants.

VIII. New Suggested Detection Approach

i. Gravitational Redshift Measurements

The gravitational field of neutron stars engenders a unique environment that might unveil the presence of dark matter. Accumulation of dark matter within these stars could alter their gravitational field, thereby affecting the observed redshift of emitted radiation. A discernible shift in the emitted spectrum, as compared to theoretical expectations, could offer a tantalizing glimpse into dark matter interactions. Noteworthy explorations of gravitational redshift in neutron star spectra are explained in the works of Tang et al. [29]. In which a new novel approach was proposed to measure the mass of Isolated neutron stars (INS) as there have not been any previous measurements of any INS only ones in binary systems.

Tang el al approach depends on using the equations of state (EoS) constrained with gravitational wave (GW) data, nuclear experiments, and the maximum mass of nonrotating NS. This is because, with the constrained EoS, we can map a series of mass–radius (M–R) points that were obtained by solving Tolman–Oppenheimer Volkoff (TOV) equations to the gravitational redshifts (zg) and the masses of NS.

To infer the masses of INSs using the EoS sets predicted by GW data, nuclear experiments, and the bounds on MTOV, the measured gravitational redshift of an NS is transformed to its compactness; therefore, the neutron-star mass can be obtained once the relationship of M(R) is known. For each EoS, varying the central pressure or pseudo enthalpy, a curve could be drawn in the M–R plane. Then, it is straightforward to have the zg–M curves using:

$$z_g = \frac{1}{\sqrt{1 - \frac{2GM}{RC^2}}} - 1 \tag{4}$$

where G is the gravitational constant and c is the speed of light. Then, 1000 values of zg from the measured distribution of gravitational redshift.

IX. Discussion

The findings presented in this paper provide a comprehensive overview of interactions between dark matter and neutron stars, and the primary approaches used to detect it. In addition, it also highlights the challenges facing these approaches. Moreover, a novel detection method is introduced utilizing the gravitational redshift measurements to detect the presence of dark matter; however, due to a lack of precise data and instrument limitations, this finding should be interpreted with caution. Therefore, in-depth analysis of the preliminary redshift measurements should be conducted on real-life neutron star examples whenever the data is available. Notability these analyses are recommended to be conducted on neutron stars of binary systems for their numerous advantages that have been discussed above.

In summary, this paper not only provides a comprehensive synthesis of knowledge regarding the intricate relationship between dark matter and neutron stars but also introduces a cutting-edge detection method. While the gravitational redshift approach holds promise, it is vital to acknowledge its limitations and invest in the meticulous analysis of real-world data to further our understanding of this complex interplay.

X. Conclusion

The enigma of dark matter continues to captivate scientists while navigating its elusive realm constituting a significant part of our universe. As neutron stars serve as potential sites for dark matter interaction, our pursuit encompasses direct and indirect dark matter detection methods like neutrino emissions and gamma-ray signatures. Yet, existing approaches face significant challenges.

Therefore, our novel approach delves more into the intricate task of unveiling dark matter within neutron stars, leveraging advanced technology and innovative methods, concluding that the presence of dark matter in neutron stars is detectable through the utilization of gravitational redshift measurements as they offer a distinct perspective inferring the masses of INS using the EoS sets predicted by GW data, and nuclear experiments. Which results in a difference between the expected mass of INS and the mass calculated through gravitational redshift measurements. However, due to a lack of precise measurements and instrument limitations, applying these equations remains unfeasible. So, further research and advancements are required to overcome these obstacles and realize its full potential.

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