Dark Energy and its Influence on Cosmological Aspects

Belal Bahaa, Omar El-Wakeel, STEM High School for Boys – 6th of October

Abstract

Cosmologists are facing mysteries. Some of which is dark energy, the overall shape of the universe, the density of the universe, etc. The discovery of the current fast expansion of space fundamentally altered cosmology and the outlook for all life. It appears that neither the hopeful hypotheses of a universe that is expanding continuously (but at a slower rate) nor one that is collapsing exist anymore. The result appears to be very bleak. This paper aims to show what have the scientific field achieved in these topics and the history behind how did these mysteries arise. This paper discusses the latest research on dark energy and its influence on our understanding of the universe. The dark energy, which is thought to be described through its equation of state and other powerful equations, may be the key to some of the mightiest questions in the history of humanity. Our analysis used prominent cosmological models to explain the ambiguity of dark energy and its properties, the overall shape of the universe and whether it has positive, negative, or flat curvature, and how the density of the universe constituents influences the progression of the universe. We concluded through this paper that dark energy's ambiguity influences our understanding and conceptualization of our universe.

I. Introduction

Astronomers once held the view that the universe was static and unchanging. They felt that while there was local movement like the earth's rotation around the sun and the moon's rotation around the earth, on a cosmic scale everything was suspended in space and that there was no beginning or end to the world. However, if they were correct, the cosmos would have shrunk due to the gravitational pull of galaxies. On the basis of this, Einstein came to the conclusion that there was a second force, which he termed "The cosmological constant." According to Einstein, this force would sustain a fixed universe by being equal and opposite to the force of gravity. Later it was found out that the universe was actually expanding due to something called “dark energy”. It was thought later that the cosmological constant was a failure, but it turns out it might be the solution.

When scientists discovered that the universe's expansion was actually speeding up, they linked it to vacuum energy (dark energy or cosmological constant). Currently, physics can't fully explain dark energy; however, scientists have been able to derive several properties about it. Dark energy is a vacuum energy and is responsible for the universe's accelerating expansion. Dark energy is typically modeled as a perfect fluid with negative pressure [1]. Furthermore, scientists believe that its density is constant (written equivalently as $\rho \propto a^0$, where we will use this notation later on [1] [3]).

Our objective is to show the latest research in dark energy and its influence. This research will specifically analyze the ambiguity of dark energy, go into great detail about how it affects the future of the universe [2], and how it influences the critical point of the universe and the geometrical shape of space curvature.

II. Dark Energy: What exactly is it?

In the early twentieth century, astronomers thought that the universe was static. In the past, this was the accepted theory when Einstein was building his general theory of relativity [12]. After formulating his gravitational field equation, he found that these equations describe a universe that must be shrinking due to the gravitational force of matter. To solve this problem, he added the cosmological constant, which was supposed to be like "anti-gravity" [1] and stop the universe's shrinkage.

Later on, scientists proved that the universe was expanding [14]. Consequently, Einstein removed the cosmological constant and called it his "biggest blunder." This was the case until scientists proved that the expansion was accelerating by observing type Ia supernovae [4]. This insane discovery showed that there must be some energy that causes this accelerating expansion, which is known now as dark energy. The value of dark energy is not known till now. Theories suggest that the cosmological constant is the constant of integration when one integrates the local energy and momentum. This would imply that no one can assign a certain value of the cosmological constant. Therefore, experimental calculations are crucial to try and evaluate the value of the cosmological constant.

In the scientific field, dark energy is the same as the cosmological constant[13]. This is why the cosmological constant was added back to the gravitational field equations by adding the term $\Lambda g_{\mu\nu}$ [1]. The current physics can't fully explain dark energy; however, some properties are discussed below.

Dark energy was discovered to make up approximately 76% [5] [15] of the universe using theoretical and experimental data. This is a consequence of a flat universe, which is discussed in the next part. Furthermore, dark energy has a constant density. This is consistent with our understanding of dark energy since it is thought to be a property of the vacuum itself. It has a much smaller density compared to the average density of matter in the universe, but it represents a significant percentage of the universe since it is uniformly distributed in it. Generally, dark energy does not interact with any of the fundamental forces other than gravity, making it very hard to detect along with the fact that it is not very dense.

Moreover, dark energy has negative pressure. This is consistent with the fact that it causes the universe's expansion. Dark energy is represented through the energy-momentum tensor $T_{\mu\nu}$ of a perfect fluid:

$$T_{\mu\nu} = \begin{bmatrix} \rho c^2 & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{bmatrix}$$

Where the term ρc^2 represents the energy density and $-p$ represents its pressure. Moreover, all these zeros represent that dark energy has no momentum flux, momentum density, energy flux, or shear stress.

The energy-momentum tensor describes a fluid that is at rest. This arises the question of in which frame of reference is the universe at rest. Also, general and special relativity demands that there should not be an "objective" frame. It turns out that there is a frame known as the cosmic rest frame in which the universe could be described to be at rest. This model is concerned with the Doppler shift of the cosmic microwave background radiations, which is also known as "CMBR". These radiations are found everywhere in the universe and has an origin that starts since the big bang. Since they are constantly moving through space, they are shifted. A frame where no shifting and the CMBR has constant frequency is considered a cosmic rest frame. This is also the frame where if you "zoom out", the universe would look the same and the average speed of the universe may be regarded as zero.

This frame may be confused with the aether, which was proposed a long time ago to explain in which frame did Maxwell's equations work. However, the aether was rejected by the scientific community later on because many experiments deduced that there was no aether. These two concepts may look alike for the first moment; however, there is a subtle difference between them. The aether was proposed to be considered the frame in which Maxwell's equations work. It was considered to be the medium through which light pass through. Later on, aether was rejected since it didn't exist. Special and general relativity, on the other hand, rejects the aether and claims that the laws of physics are applicable in any frame. The cosmic rest frame, however, isn't strictly about applying physical laws but about the distribution of matter and radiation in the universe. There may exist several frames like that, but they are collectively known as a cosmic rest frame.

All these properties result in different models for the overall shape of the universe and its evolution over time. The overall curvature of the universe could solve many unsolved problems and unlock a better understanding of the early universe and how we got here.

III. Space Curvature: What does our universe look like?

As known to the world today, matter can curve space, creating gravity. Scientists before the 20^{th} century didn't imagine that gravity is caused by massive objects bending space. In the 20^{th} century, Einstein formulated his theory of relativity, showing that energy and mass bend space-time.

Cosmology is generally based on the cosmological principle [7], which states that the universe is homogeneous and isotropic. The cosmos is homogeneous on a broad scale. Homogeneous simply indicates that there are no distinctive locations and that every area of the cosmos is essentially identical to every other area. The universe is plainly exceedingly lumpy and inhomogeneous at small. However, the cosmos is at last uniform on scales greater than superclusters and voids. A sphere with a radius of 100 Mpc has the same average density of matter as any other sphere with a similar size.

The cosmos is isotropic on a large scale. Isotropy simply refers to the fact that no matter which way you look; you see the same perspective since the cosmos is essentially isotropic. The universe is clearly anisotropic at tiny sizes; there are favored directions. However, the universe is isotropic on scales greater than superclusters and voids.

In his theory of general relativity, Albert Einstein explained that on tiny scales, space is "dimpled" by enormous objects like stars, galaxies, or clusters of galaxies. The premise of homogeneity and isotropy, however, requires space to maintain a constant average curvature throughout at large scales.

There are three acceptable theories for the overall curvature of the universe that obey the cosmological principle. These are positive curvature (Sphere shaped or S^3 geometry), negative curvature (Hyperbolic shaped or H^3 Geometry), and zero curvature (Flat).

i. Spherical Universe

First, positive curvature in space-time is implied by positive mass. All of the objects in the universe have an effect on space-time but most of them are too insignificant to notice. On the other hand, massive objects like the stars or the planets have a noticeable effect.

Figure (1) shows the effect of a massive object on the space-time and the surrounding objects. The shape of the curvature illustrated in figure (1) is a positive curvature and it is caused by positive mass-energy.

If the average curvature of the universe is positive, the universe would be a large sphere and the laws of basic Euclidean geometry won't apply. The angles of a triangle drawn on a positively curved universe would add up to more than 180 degrees. Positive curvature would also imply that the universe's expansion will be reversed by the gravitational force of matter.

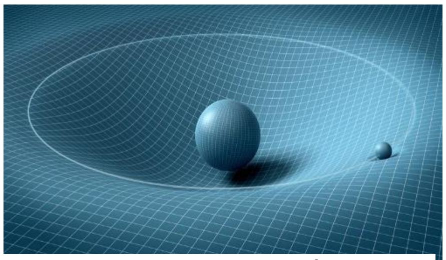

Figure (1) shows the curvature of space-time by mass

ii. Hyperbolic Universe

` Second, negative curvature in space-time is implied by negative mass and energy. In a negatively curved space, rules of basic Euclidean geometry don't apply. Three angles of a triangle drawn on a negative curve wouldn't add up to 180 for example as shown in figure 2. A hyperbolic universe would imply an infinite expansion since matter won't be able to pull the expansion back.

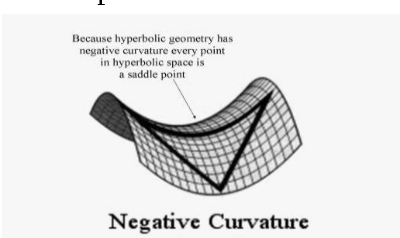

Figure (2) shows a hyperbolic universe (negative curvature)

iii. Completely Flat Universe

Third, as the positive curvature is implied by positive mass and energy, and negative curvature is implied by negative mass and energy, zero curvature is implied by 0 mass and energy and it looks like a 2-D plain. It's literally the absence of mass and energy. The laws of basic geometry apply in an average flat curved universe and it would have infinite area and volume. A flat universe would also imply infinite expansion, and matter won't be strong enough to dominate this expansion. A flat universe with dark energy is generally modeled by the ΛCDM model [8][10].

The Hyperbolic and spherical geometries are also known as de-sitter and anti de-sitter space respectively. They were named after Willem de Sitter who worked with Albert Einstein on the problem of the curvature of the universe. Both spaces are essentially the same, where both of them are maximally symmetric Lorentzian manifold, which is a space-time where no point can be distinguished in any manner from another. The difference between them is that a hyperbolic universe implies that in the absence of matter or energy, the curvature of space-time is negative. This also corresponds to a negative cosmological constant. On the other hand, a spherical universe implies positive curvature and a positive cosmological constant in the absence of matter or energy. However, recent researches have shown that the cosmological constant is positive, rolling out a hyperbolic universe.

There also exist other geometrical shapes that obey the cosmological principle, such as a torus; however, these geometries are far too complicated and sometimes incomplete. All these curvatures have been acceptable until new experimental data have showed that the universe is approximately flat [6], showing that the ΛCDM may represent an understanding of the real universe. Furthermore, the curvature of the universe is mainly influenced by the average density of the universe.

iv. The Lambda-CDM model (ΛCDM)

A crucial Big Bang cosmology prediction was validated by the 1965 discovery of the Cosmic Microwave Background. After then, it became widely acknowledged that the cosmos began in a hot, early condition and has been expanding ever since. The forms of matter and energy in the cosmos and, in particular, whether the total density is above or below the so-called critical density, discussed in part IV, affect the pace of expansion. Pure-baryonic models received the majority of attention in the 1970s, but they faced significant difficulties in describing how galaxies form given the CMB's (Cosmic Microwave Background) low anisotropies (upper limits at that time). Models with critical densities were driven by the hypothesis of cosmic inflation at the beginning of the 1980s as it was understood that this issue could be overcome if cold dark matter (CDM) predominated over baryons. The majority of research in the 1980s concentrated on cold dark matter, which successfully formed galaxies and clusters of galaxies with a critical density of matter of about 95% CDM and 5% baryons. However, problems persisted, not the least of which was that the model called for a Hubble constant that was lower than preferred by observations and under-predicted observed large-scale galaxy clustering. With the COBE's 1992 finding of CMB anisotropy, these challenges became more acute, and a number of solutions, such as Lambda CDM and mixed cold and hot dark matter, were given serious attention.

After the observations of accelerating expansion in 1998, the Lambda CDM model was adopted as the norm, and it was soon confirmed by additional observations: in 2000, the BOOMERanG microwave background experiment determined that the total (matter + energy) density was close to 100% of critical, while in 2001, the 2dfGRS galaxy survey determined that the matter density was close to 25%; the significant discrepancy between these two values supports a positive cosmological constant or dark energy. The model has continued to receive support and improvement because too far more accurate observations of the microwave background from WMAP between 2003 and 2010.

Many facets of the CDM model are currently the subject of active research, both to improve the parameters and perhaps find aberrations. The nearly scale-invariant spectrum of the CMB perturbations and their image across the celestial sphere are thought to result from very small thermal and acoustic irregularities at the point of recombination. Additionally, the CMB has no explicit physical theory for the origin or physical nature of dark matter or dark energy. The majority of astronomers and astrophysicists back the CDM model or closely related models, but leading critics Milgrom [10], McGaugh, and Kroupa attack the dark matter portions of the theory from the viewpoint of galaxy formation models and back the alternative MOND theory, which calls for changing the Einstein Equations and the Friedmann Equations as seen in proposals like MOG theory or TeVeS theory. In an effort to explain dark energy or dark matter, theoretical physicists have also proposed f(R) gravity, scalar-tensor theories, brane cosmologies, the DGP model, and galileon theories as cosmological alternatives to Einstein's general relativity and in an effort to explain dark energy or dark matter.

On broad dimensions (scales larger than galaxies, up to the observable horizon), comparison of the model with observations is very effective; nevertheless, on sub-galaxy scales, it may predict too many dwarf galaxies and too much dark matter in the innermost regions of galaxies. It is not yet apparent whether the issue is the simulations, unusual characteristics of dark matter, or a more severe flaw in the model because these small scales are more difficult to resolve in computer simulations. In addition, the physical baryon density, physical dark matter density, dark energy density, scalar spectral index, curvature fluctuation amplitude, and reionization optical depth are the six characteristics upon which the lambda CDM is founded.

IV. The density of the universe: Cosmic evolution

The gravitational field equation in general relativity implies that the universe should have collapsed due to the gravitational force of matter found within. Thus, it became cosmologists' goal to determine how the density of the universe's constituents evolved during its expansion. First, we have to understand how the density of matter, radiation (Cosmic Microwave Background Radiation), and dark energy evolved.

First, let us discuss the equation of state of dark energy. The equation of state is a renowned idea in thermodynamics and chemistry. Its main cause is to relate state variables of some process. The simplest known state equation is $PV = nRT$, which is the ideal gas law. Cosmologists tried to predict what is the equation of state of Dark energy is. There are numerous possibilities for the equation of state; however, the simplest choice is assuming that the equation of state for the universe is $p = w\rho c^2$. This equation relates the pressure p to the energy density ρc^2, which is defined as the energy per unit volume. This equation of state is under the assumption that the universe is a barotropic fluid, which is a fluid that has its density as a function of pressure only. Furthermore, these two variables are related by the constant w. We can try different w values. Starting with zero, the described matter has no pressure. This is known as dust, which is a type of matter that exerts no pressure on its surroundings. Another way to use this equation of state is to derive an equation that can describe electromagnetic radiation. It is very complicated to derive. So, for simplicity, we can start by knowing that the trace of the energy momentum tensor of electromagnetic radiation is equal to zero. The trace is the sum of terms on the main diagonal of some matrix. Therefore,

$$\rho c^2 - 3p = 0$$

This gives us the result that $w = \frac{1}{3}$. Now we can try to assume that the value of w is equal to -1. If we do the substitution, we find that the equation is reduced to a term that is similar to the cosmological constant term in Einstein's field equations

By substituting the equation $p = w\rho c^2$ into a variation of Friedmann's equations [1]:

$$\dot{\rho} = -3\frac{\dot{a}}{a}(\rho + \frac{p}{c^2})$$

We arrive to an equation that relates w with the density:

$$\rho(a) = Ka^{-3(1+w)}$$

Where K is just a constant. For record, the w values are known for all three entities of our interest, where matter has a w value of 0, radiation has a w value of $\frac{1}{3}$, and dark energy has a w of -1. By substituting, we get the following important relations:

$$For\ Matter: \rho(a) \propto a^{-3}$$

$$For\ Radiation: \rho(a) \propto a^{-4}$$

$$For\ dark\ energy: \rho(a) \propto a^{0}$$

These results imply lots of information about the universe's evolution. After the big bang, the temperature of the universe was so high that all matter existed in ionized forms. Therefore, the universe was originally dominated by radiation; however, its dominance faded after approximately 47,000 years. This result makes complete sense since its density varies with a^{-4}. This relation may seem counter-intuitive at first, but it makes complete sense. When the universe expands, its volume increase. Therefore, its density should vary with a^{-3}; however, we have to consider the Doppler shift that occurs to these radiation, which indeed varies with a^{-1}. Therefore, these two factors should drop its density to the a^{-4}.

Later, matter dominated over radiation, which varies with a^{-3}. This relation is analogous to the radiation mentioned above. However, we don't consider the Doppler effect factor since matter isn't affected by that.

The marvelous solution is that dark energy's density is constant. This answer is also intuitive because dark energy is a property of space. This implies that when the universe expands, dark energy increases as well. This actually violates the law of conservation of energy, but the law of conservation of energy is actually not fulfilled on cosmic scales.

This arises the question of whether matter can overtake dark energy, and the universe shrinks. This is answered by the critical mass density of the universe.

First, the universe would neither expand nor contract if the energy of motion (Kinetic energy) balances the gravitational potential energy [9] . This can be written as:

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

Since Hubble's law states that $v \approx H_o R$, we can make this substitution in the aforementioned equation

$$\frac{1}{2}m(H_o R)^2 \approx \frac{GMm}{R}$$

$$\frac{1}{2}H_o^2 \approx \frac{GM}{R^3}$$

$$\rho_c = \frac{M}{{}^4\!/_3\,\pi R^3} \approx \frac{3H_o^2}{8\pi G}$$

Taking Hubble's Constant $(H_0) \approx 70.8 \pm 1.6\ km\ s^{-1}\ Mpc^{-1}$ and the Newtonian gravitational constant $G = 6.67 \text{ x } 10^{-11}\ N\ m^2/kg^2$, we get that the critical mass density (ρ_c) to be approximately $10^{-26}\ kg/m^3$. This means that if the mass density is less than this value, the universe expands, and if it is greater than that, the universe contracts.

Cosmologists prefer to describe density using density parameter (Ω_o), which represents the ratio between the combined average mass density (ρ_o) and the critical mass density (ρ_c). An open universe (Also known as negative curvature or hyperbolic) has an Ω value between 0 and 1. A flat universe has an omega value of 1, and a closed universe (Also known as positive curvature or spherical) has an omega value that is greater than 1. The current mass density is roughly 2.4 x10^{-27} . Dividing them, we get the ratio of mass-density parameter (Ω_m) is 0.24. However, experimental evidence showed that the universe is flat or nearly flat. This implies that the omega value must be (nearly) equal to 1.0.

Cosmologists were baffled by this problem, which is that the matter of the universe accounts for only 24 percent of the universe. Cosmologists, later, showed that the dark energy (cosmological constant) accounts for the rest (76 %) [11], which is represented by (Ω_Λ). The experimental and theoretical evidence suggested that the values of (Ω_m), (Ω_o), and (Ω_Λ) [5] are as follows:

$$(\Omega_m) = 0.241 \pm 0.034$$

$$(\Omega_\Lambda) = 0.759 \pm 0.034$$

$$(\Omega_o) = 1.02 \pm 0.02$$

V. Conclusion

In conclusion, we observed that the physics behind dark energy isn't clearly figured out. Moreover, we are unable to prove that there isn't another physics applied to another cosmology that agrees with the cosmological tests just as well. Of course, this also holds true for the entire field of physical science. Certain aspects of physics are so thoroughly tested that they make for remarkably accurate approximations of how the world actually is. In cosmology, the network of tests is much less dense but, as we have tried to show, by no means insignificant, and it is getting tighter. The question of how to test general relativity theory on cosmological scales was not widely discussed ten years ago. That was due in part to the theory's apparent plausibility and most definitely a result of the lack of sufficient supporting evidence. The empirical situation is much better now.

Until recently, it made sense to take into account the limits on one or two cosmological parameters while maintaining the remaining parameters at "acceptable" values that aid in our comprehension of the measurements' intended purpose. However, the current and quite sane tendency is to take into account the combined fits of numerous parameters to the entire set of observations. A truly satisfying cosmic test will also include parametrized deviations from conventional physics that are projected to cosmological scales in the parameter set.

Community reactions to new empirical findings are not necessarily linearly correlated. The Einstein-de Sitter model's popularity persisted into the 1990s for a longer period of time than it makes sense to us, and the transition to the now-standard CDM cosmological model, which includes flat space sections, nonbaryonic cold dark matter, and dark energy, is possibly more abrupt than is justified by the developments in the evidence. According to our review, there is currently a strong scientific argument that the matter density parameter is roughly 0.24 and that roughly three-quarters of that is not baryonic. The arguments for dark energy and the CDM model are also important, but they are complicated by observational questions about whether we have a good understanding of how structures form. We anticipate that these clouds will quickly disperse, however, and that, given the track record, fresh clouds will most likely be discovered in the standard model of cosmology ten years from now.

Over the past eight decades, there have been steady advancements in the equipment used to conduct cosmological tests, from telescopes to computers; in the theoretical underpinnings of the tests; and in the learning curves for doing so. The facts are clear: the foundation of cosmology is far more solid than it was ten years ago, and in ten years, the foundation will definitely be substantially stronger. Einstein's cosmological constant and its contemporary counterpart, dark energy, have been discussed across a wide range of physics and astronomy-related topics for the majority of the last eight decades, at least in some circles. Undoubtedly, many of these problems have been found more than once. However, based on our observations, these concepts frequently endure for a long time with little attention and occasionally with low fidelity. As a result, the public has been very well prepared for the current evidence of dark energy detection. And for the same reason, regardless of the results of the current work on the cosmic tests, we think that dark energy, whether constant, rolling toward zero, or possibly even increasing, will still be an active topic of inquiry, in at least some circles, a decade from now. We have no basis for predicting whether the conventional cosmology in ten years will be a straightforward elaboration of the Lambda-CDM or whether there will be more significant changes of direction, despite the fact that this much is apparent.

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VII. Appendix

Table of Variables:

Variable	Definition	Sign
Mass density	Mass to volume ratio	ρ
Scale factor	It is a function that expresses relative expansion of the universe	a
Cosmological constant	It represents the force that prevents the universe's shrinking and is the same as dark energy	Λ
Metric Tensor	It allows us to define angles and distances near points on a surface	$g_{\mu\nu}$
Energy-momentum tensor	It describes density, and flux of energy and momentum	$T_{\mu\nu}$
Pressure	Stress per unit area	p
Speed of light	Speed of light in vacuum	c
Pressure-density ratio	Pressure to density ratio	w
Change of density over time	$\frac{d\rho}{dt}$	$\dot{\rho}$
Change of scale factor over time	$\frac{da}{dt}$	$\dot{a}$
Change of density with respect to the scale factor	It represents how density is changed by changing the scale factor	$\rho(a)$